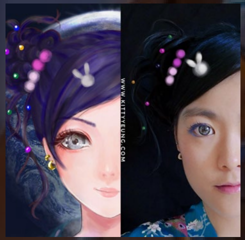
Introduction to Quantum Computing



Kitty Yeung, Ph.D. in Applied Physics

Creative Technologist + Sr. PM
Microsoft

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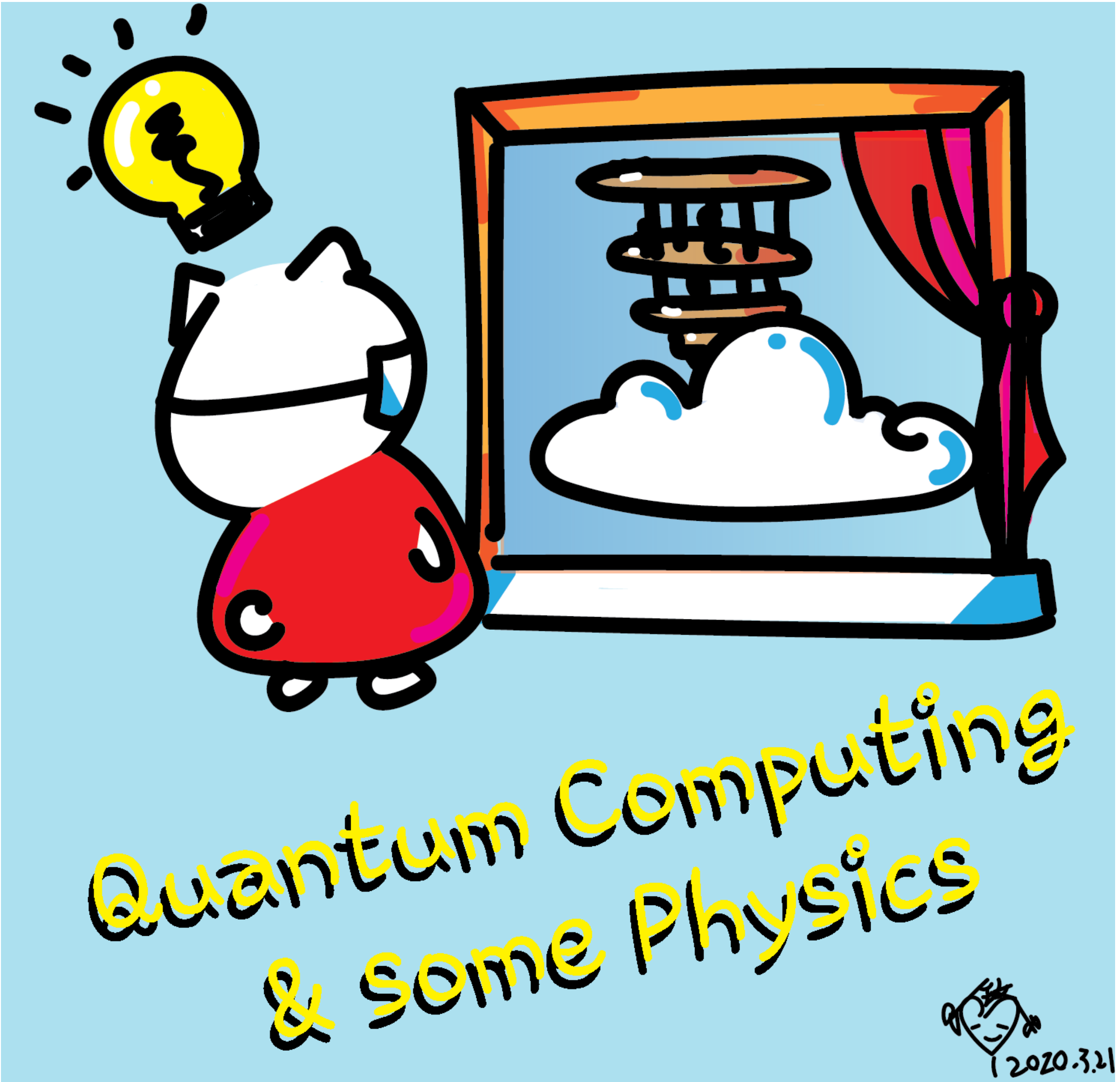
@KittyArtPhysics
@artbyphysicistkittyyeung

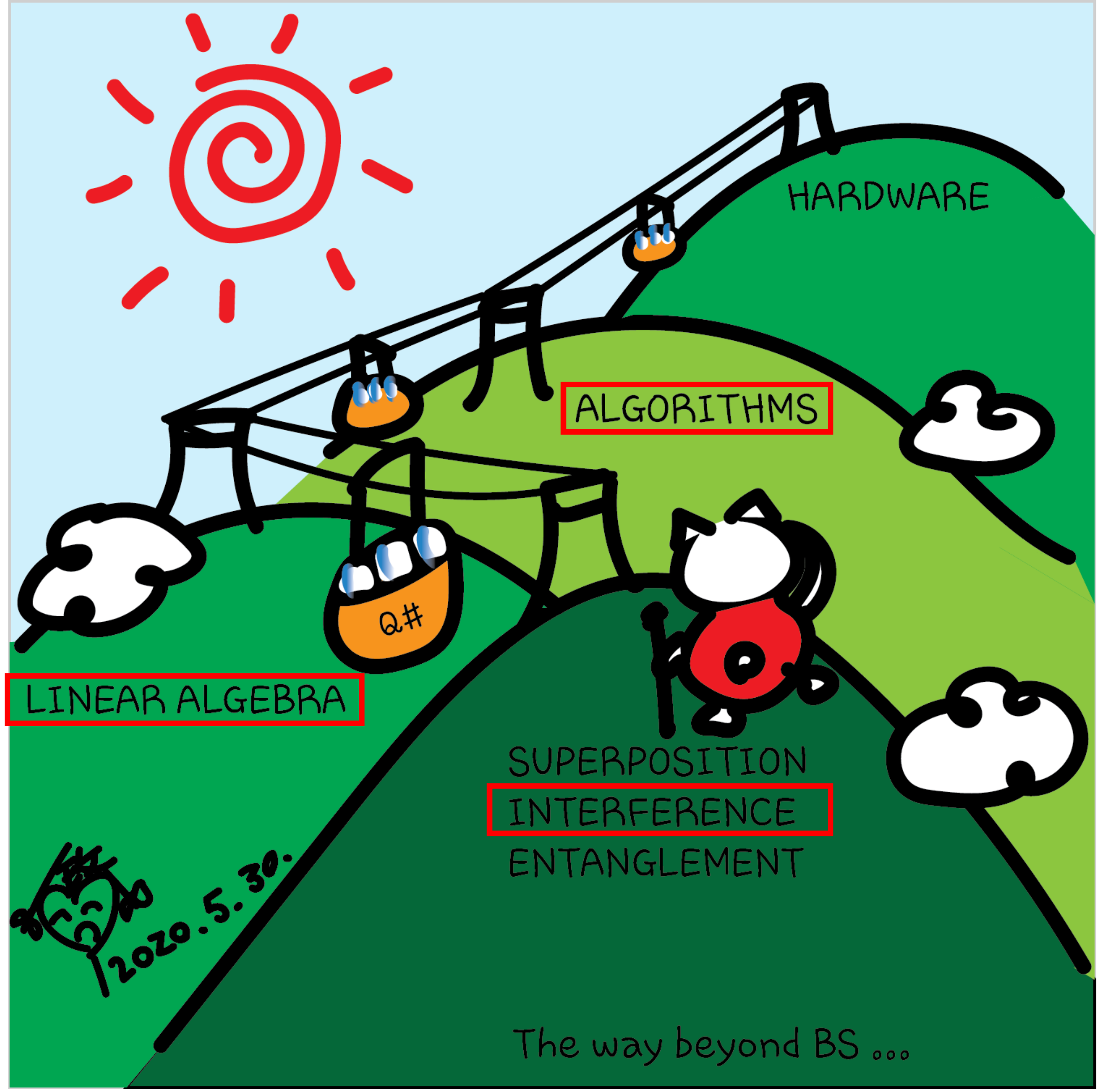
June 14, 2020
Hackaday, session 11
Other communities, session 3

Class structure

- Comics on Hackaday – Introduction to Quantum Computing every Sun
- 30 mins – 1 hour every Sun, one concept (theory, hardware, programming), Q&A
- Contribute to Q# documentation http://docs.microsoft.com/quantum
- Coding through Quantum Katas https://github.com/Microsoft/QuantumKatas/
- Discuss in Hackaday project comments throughout the week
- Take notes



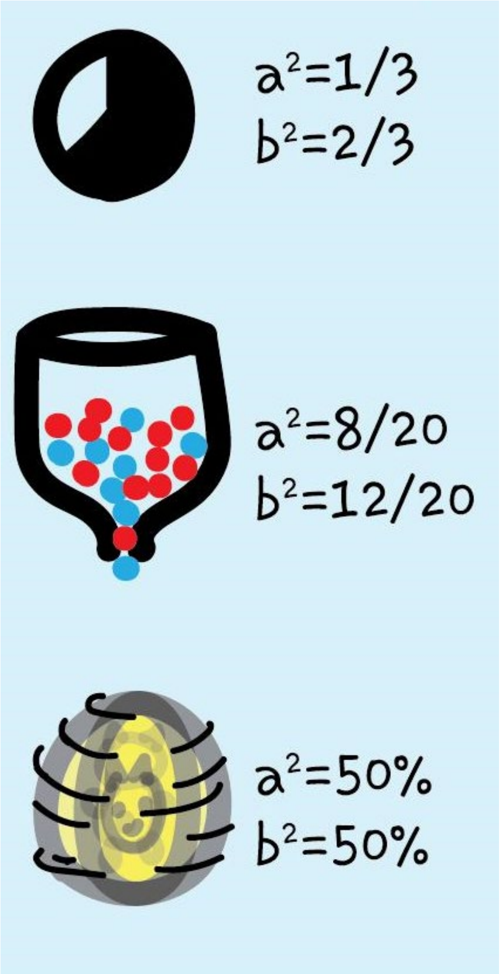
Next class


HARDWARE
ALGORITHMS
Q#
LINEAR ALGEBRA
SUPERPOSITION
INTERFERENCE
ENTANGLEMENT
2020.5.30.
The way beyond BS ...

Qubits & Superposition

$$|\psi\rangle = \begin{pmatrix} a \\ b \end{pmatrix} = a|0\rangle + b|1\rangle$$

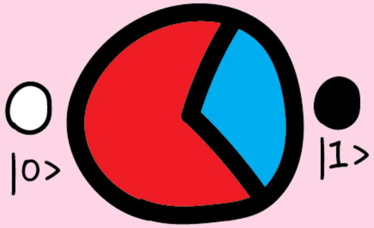
$$|a|^2 + |b|^2 = 1$$



2020.3.28.

A qubit system is all the possible configurations in superposition.

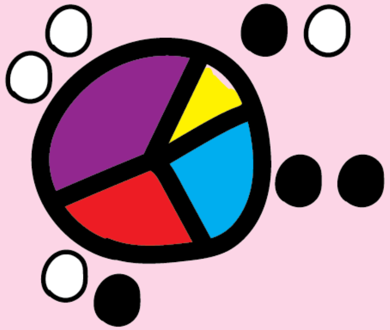
PIE CHART DENOTING PROBABILITY OF EACH CONFIGURATION



ONE QUBIT, TWO CONFIGURATIONS:

$a|0\rangle+b|1\rangle$

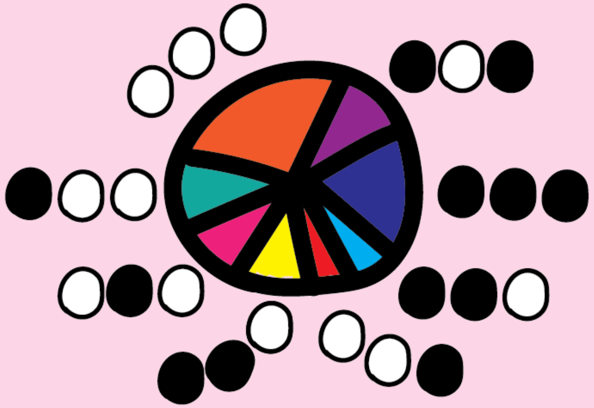
$a^2+b^2=1$ (total probability adds up to 1)



TWO QUBITS, FOUR CONFIGURATIONS:

$a|00\rangle+b|01\rangle+c|10\rangle+d|11\rangle$

$a^2+b^2+c^2+d^2=1$



...

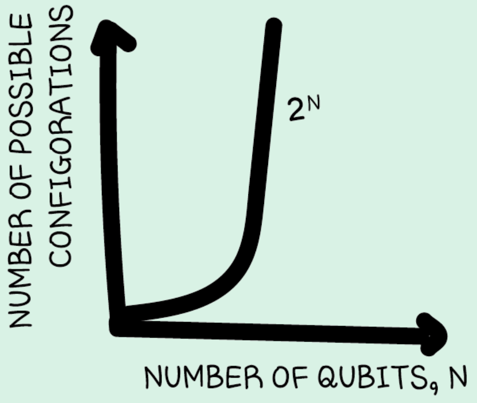
N qubits will have 2^N possible configurations in superposition!

THREE QUBITS, EIGHT CONFIGURATIONS:

$a|000\rangle+b|001\rangle+c|010\rangle+d|100\rangle+e|110\rangle+f|101\rangle+g|011\rangle+h|111\rangle$

$a^2+b^2+c^2+d^2+e^2+f^2+g^2+h^2=1$

$a|000\rangle + b|001\rangle + c|010\rangle + d|100\rangle + e|110\rangle + f|101\rangle + g|011\rangle + h|111\rangle$



Not only does the number of possible configurations grow exponentially with the number of qubits as 2^N, the number of possible combinations of amplitudes is infinite, as long as their squares – the probabilities – add up to 1.

$$|\psi\rangle = \sum_{i=1}^{N} c_i |\psi_i\rangle$$

THIS SYMBOL MEANS SUMMING ALL N TERMS FROM 1

AN N-QUBIT STATE

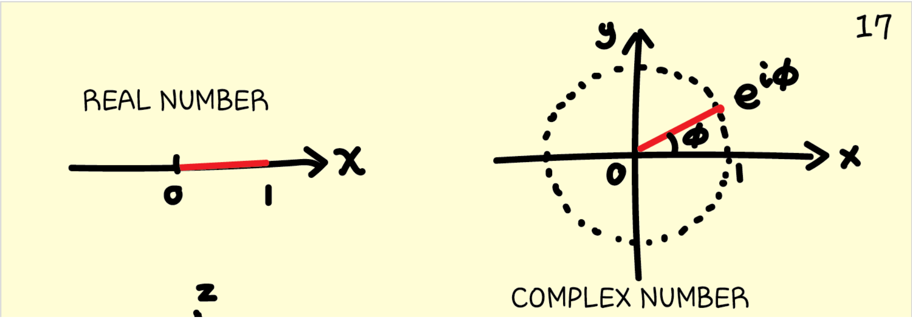
EACH POSSIBLE CONFIGURATION

The amplitude ci = a, b, c, d ...n
can be positive numbers 1, 1/2, 1/3, 1/4 ...n
or negative numbers –1, –1/2, –1/3, –1/4 ...n
(these are real numbers)
or imaginary numbers (+/–) i, 1/2i, 1/3i, 1/4i...ni
or 0.
In general they can be complex numbers (with real and imaginary parts with positive or negative signs)!

What's the consequence?

2020.4.1.

Complex numbers

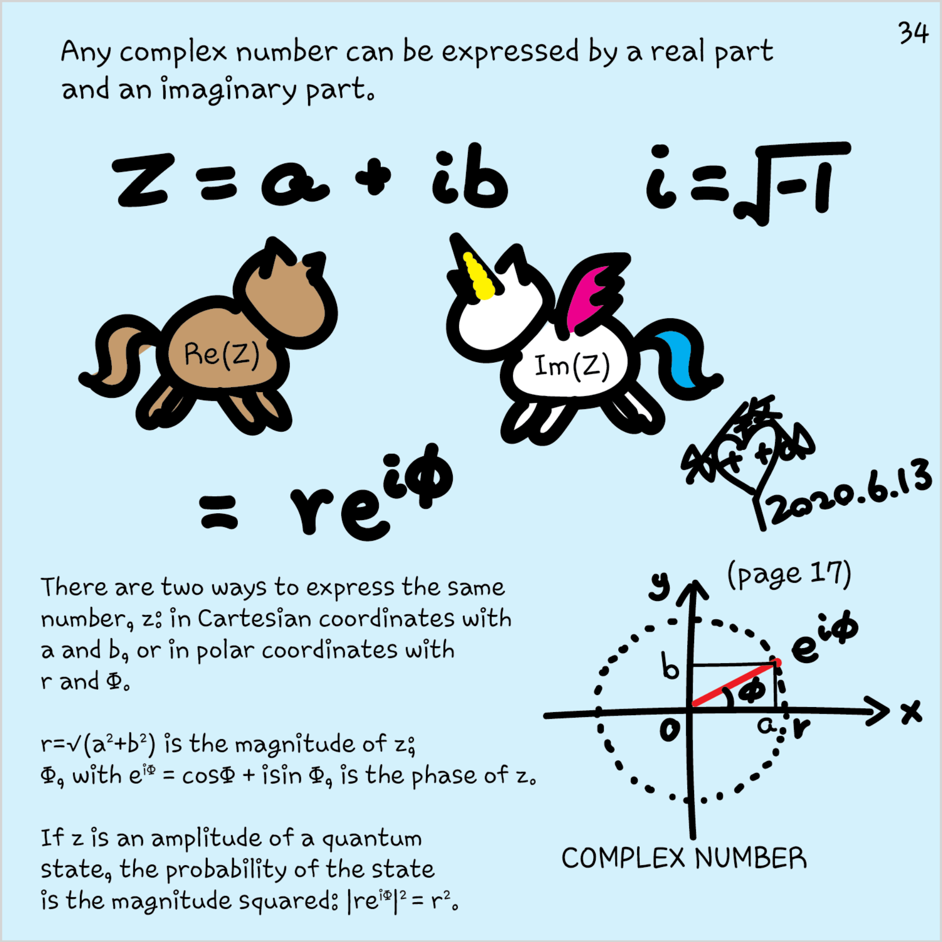


A one-qubit state $|\psi\rangle = z_0|0\rangle + \ z_1|1\rangle$ can be written as

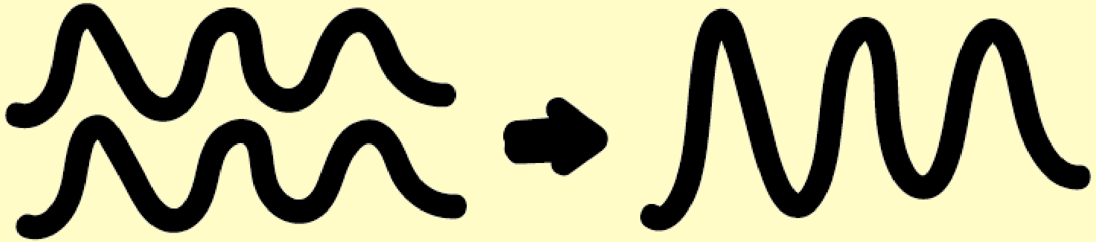
$$(a_0 + b_0)|0\rangle + \ (a_1 + b_1)|1\rangle$$

= $r_0 e^{i\varphi_0}|0\rangle + \ r_1 e^{i\varphi_1}|1\rangle$.

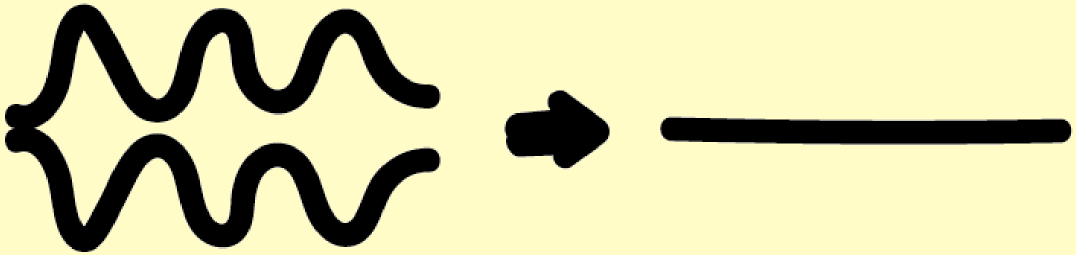
Therefore, the probability of finding state $|0\rangle$ is $\left|r_0 e^{i\varphi_0}\right|^2 = {r_0}^2$. Similarly, for state $|1\rangle$ it is ${r_1}^2$. The probability is determined by the magnitude of amplitude and is independent from phase.



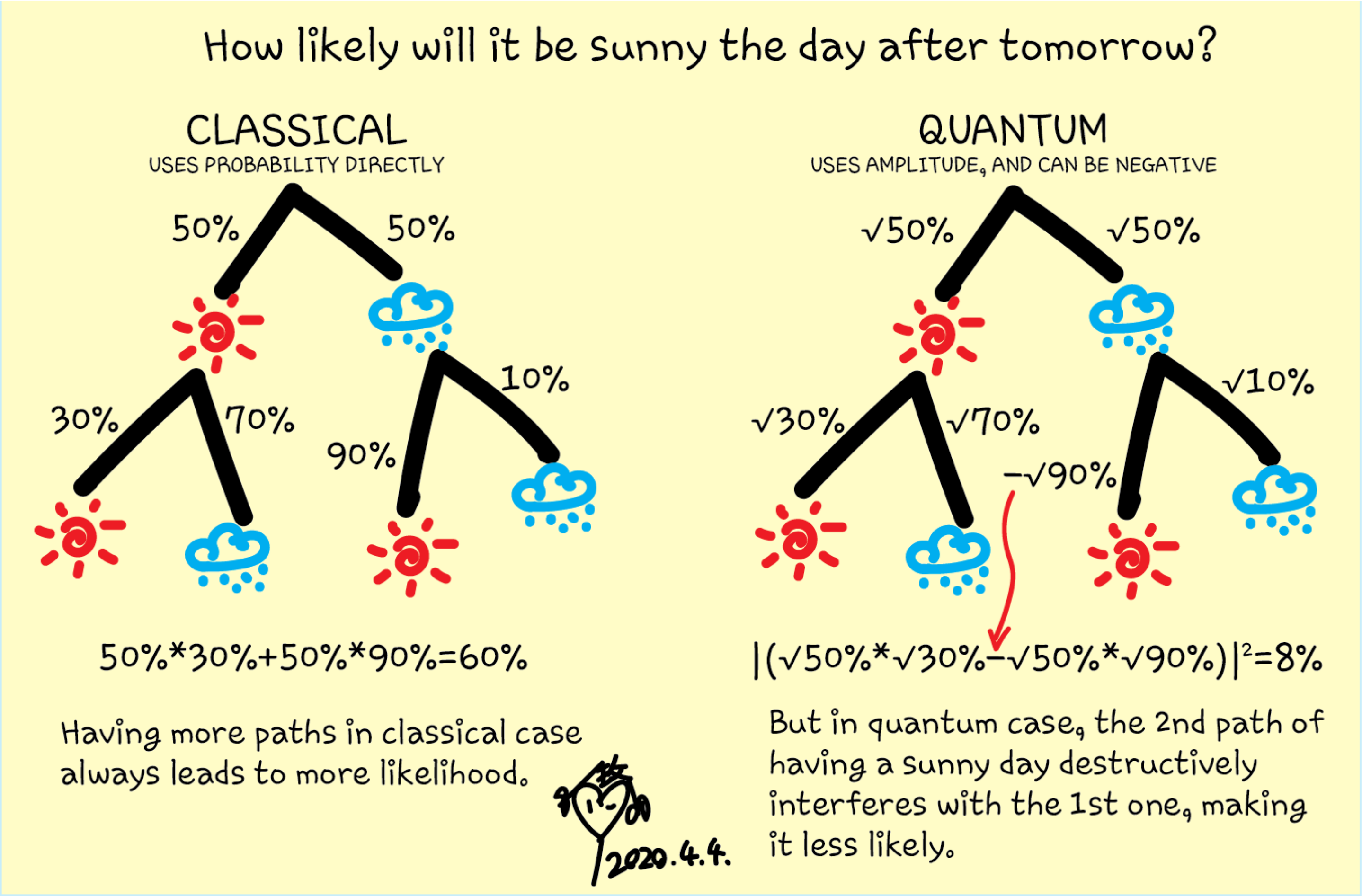
Our daily experience of amplitudes(like those of water waves, light waves, sound waves, etc.) has told us:



AMPLITUDES CAN ADD UP =
CONSTRUCTIVE INTERFERECE

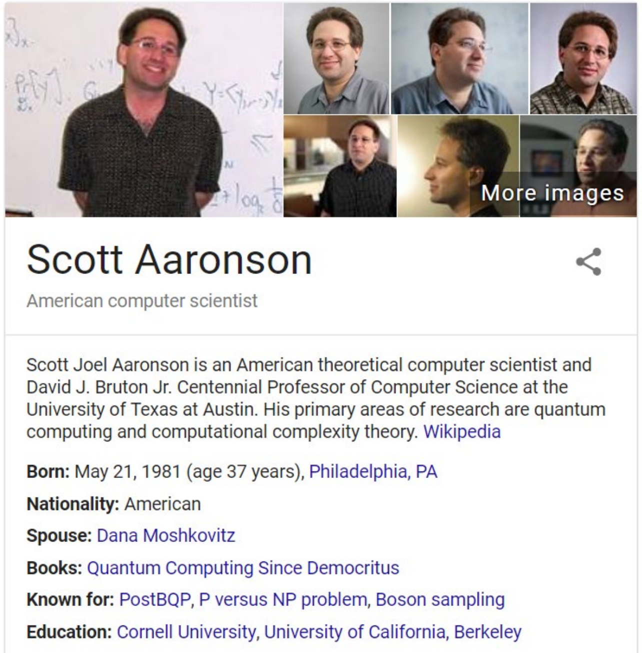


AMPLITUDES CAN CANCEL OUT =
DESTRUCTIVE INTERFERENCE



How likely will it be sunny the day after tomorrow?
CLASSICAL
USES PROBABILITY DIRECTLY
50%
50%
30%
70%
90%
10%
50%*30%+50%*90%=60%
Having more paths in classical case always leads to more likelihood.
QUANTUM
USES AMPLITUDE, AND CAN BE NEGATIVE
√50%
√50%
√30%
√70%
−√90%
√10%
|(√50%*√30%−√50%*√90%)|²=8%
But in quantum case, the 2nd path of having a sunny day destructively interferes with the 1st one, making it less likely.
2020.4.4.

Generalized probability theory



$$\sum_i p_i = 1$$

1-norm
Classical

$$\sum_i |a_i|^2 = 1$$

2-norm
Quantum mechanical

Amplitude can be positive, negative or complex

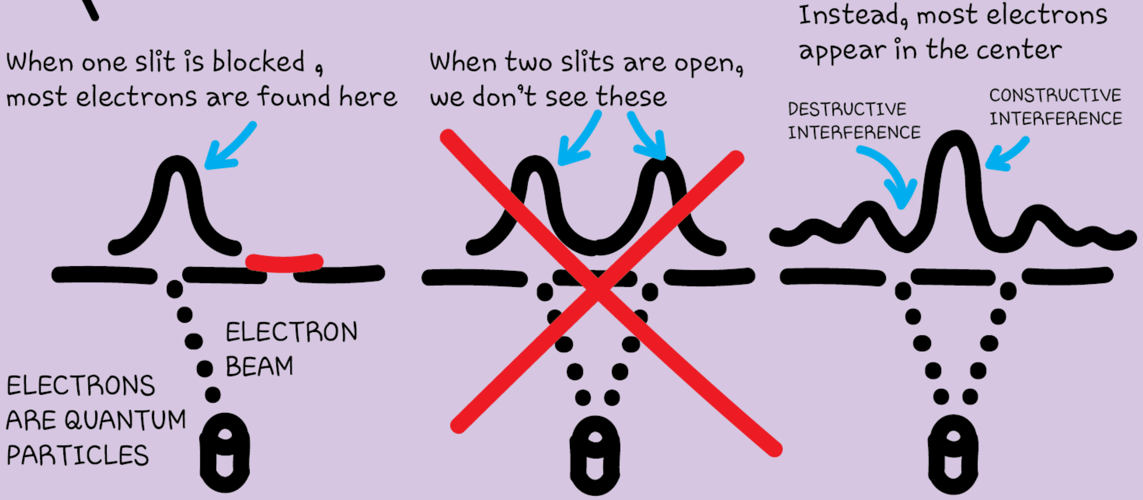
2-norm Vs 1-norm
https://www.scottaaronson.com/democritus/lec9.html

To read more rigorous mathematical derivations of the axioms in modern quantum theory:

- https://arxiv.org/abs/quant-ph/0101012
- https://arxiv.org/abs/1011.6451
- https://arxiv.org/abs/quant-ph/0104088

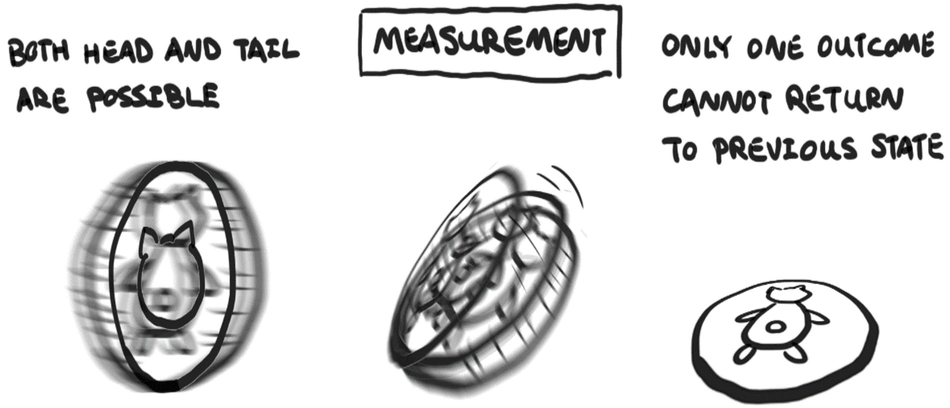
So, the things we observe (measure) are the results of interference. Possible results from constructive interference are more likely to be measured. The other possibilities cancel each other out through destructive interference.

The famous double-slit experiment is a direct manifestation of quantum interference.



Interference is one of the "strange" behaviours of quantum systems enabled by superposition. What else?

Measurement



Not reversible

$$|\psi\rangle = c_{00}|00\rangle + c_{01}|01\rangle + c_{10}|10\rangle + c_{11}|11\rangle$$

$$P = |c_{00}|^2 + |c_{01}|^2$$

If first qubit is 0

$$|\psi'\rangle = \frac{c_{00}|00\rangle + c_{01}|01\rangle}{\sqrt{P}}$$

After measurement

aka.ms/learnqc

900 XP

Create your first Q# program by using the Quantum Development Kit

48 min • Module • 8 Units

★★★★★ 4.9 (48)

Beginner | Developer | Quantum Development Kit | Quantum

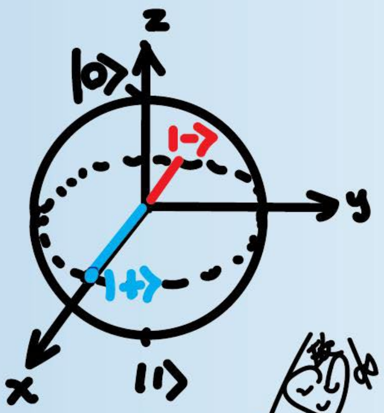
Get started with Q# programming by building a quantum random number generator.

https://docs.microsoft.com/en-us/learn/modules/qsharp-create-first-quantum-development-kit/

Q# exercise:

Option 1: No installation, web-based Jupyter Notebooks

- The Quantum Katas project (tutorials and exercises for learning quantum computing) https://github.com/Microsoft/QuantumKatas
- **Measurement**
- Tasks 1.1-1.4



Another important gate is the H (or Hadamard) gate. It changes states |0> and |1> and creates two new states in between them:

$$H|0\rangle = |+\rangle = (|0\rangle + |1\rangle)/\sqrt{2}$$

$$H|1\rangle = |-\rangle = (|0\rangle - |1\rangle)/\sqrt{2}$$

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

General rotation

In general, rotation gates, R, about an axis can be described by the angles ϕ and θ:

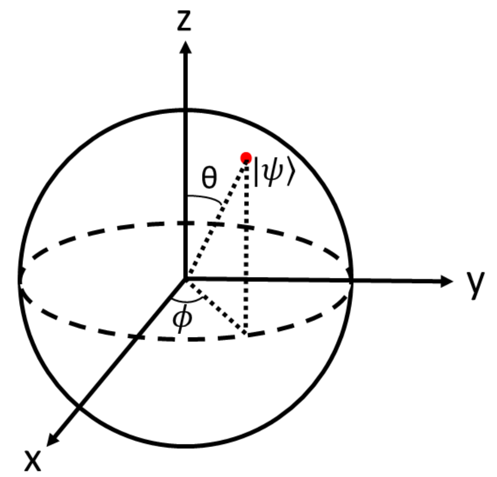
$$R_z(\phi) = \begin{bmatrix} e^{-i\phi/2} & 0 \\ 0 & e^{i\phi/2} \end{bmatrix},$$

$$R_y(\theta) = \begin{bmatrix} \cos\frac{\theta}{2} & -\sin\frac{\theta}{2} \\ \sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{bmatrix},$$

and

$$R_x(\theta) = \begin{bmatrix} \cos\frac{\theta}{2} & -i\sin\frac{\theta}{2} \\ -i\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{bmatrix}$$

$$= R_z\left(\frac{\pi}{2}\right) R_y(\theta) R_z\left(-\frac{\pi}{2}\right).$$



https://review.docs.microsoft.com/en-us/quantum/concepts/the-qubit?branch=tensor-product

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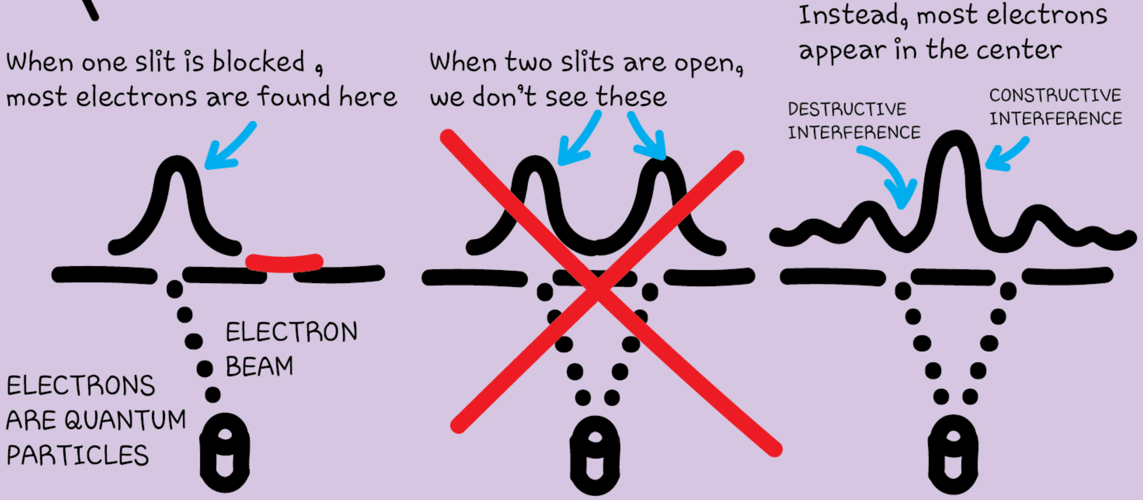

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Development of quantum mechanics (~100 years ago)

- Quantization of energy (black-body radiation – the UV catastrophe, the photoelectric effect, the Compton effect, Stern-Gerlach experiment)
- Wave-particle duality (double-slit experiment, atomic structure, de Broglie hypothesis, electron diffraction, molecular diffraction)
- Schrödinger equation

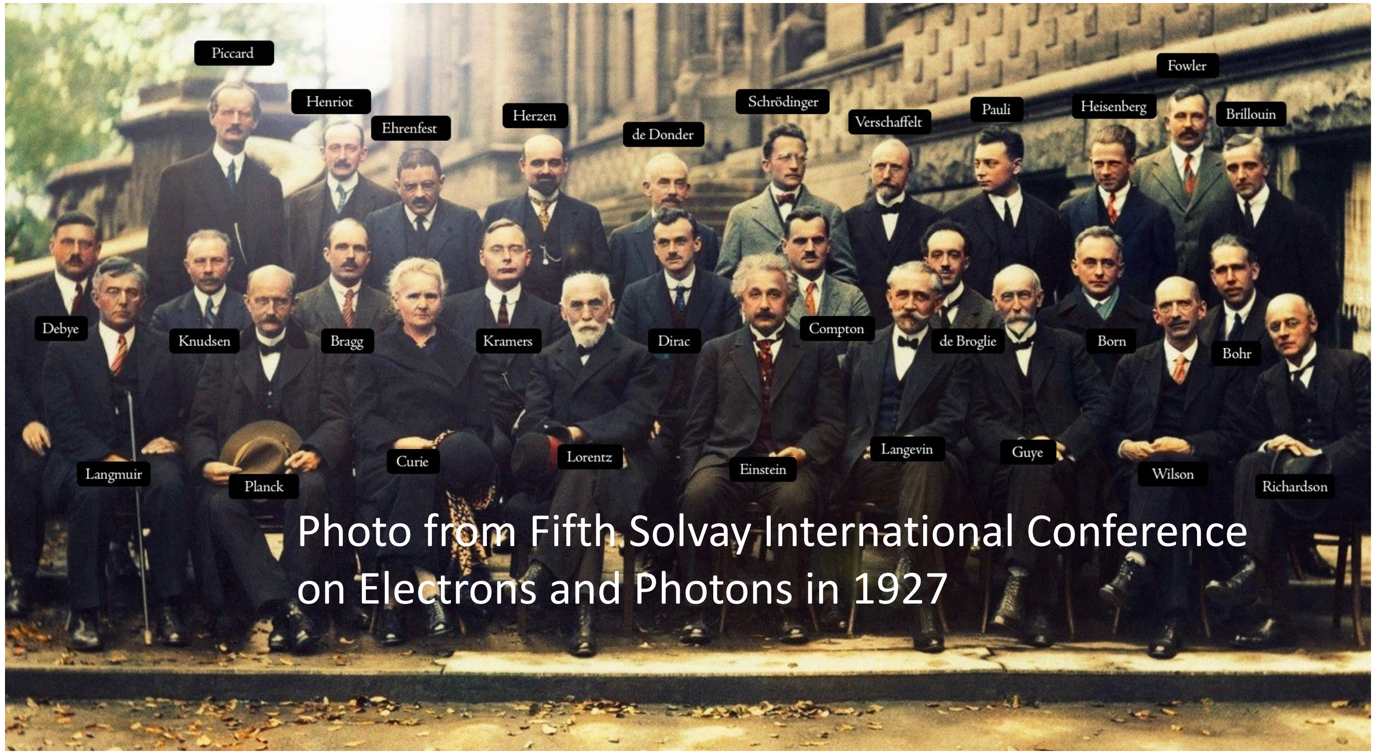
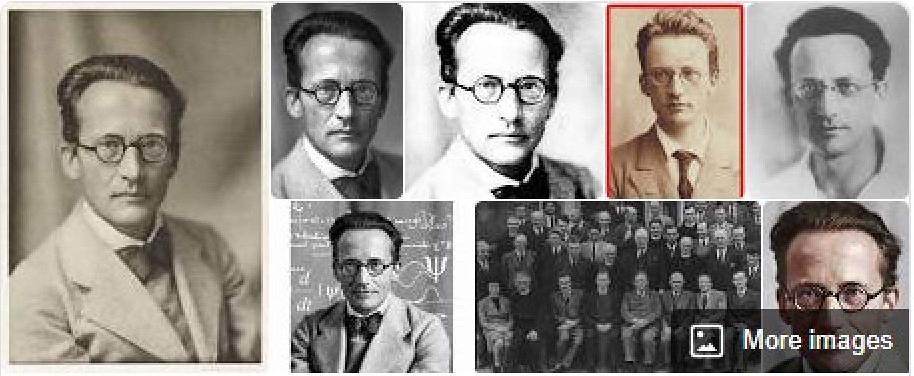


Photo from Fifth Solvay International Conference on Electrons and Photons in 1927

Schrödinger equation



Erwin Schrödinger

Austrian-Irish physicist

Erwin Rudolf Josef Alexander Schrödinger, sometimes written as Erwin Schrodinger or Erwin Schroedinger, was a Nobel Prize-winning Austrian-Irish physicist who developed a number of fundamental results ... Wikipedia

Born: August 12, 1887, Erdberg, Vienna, Austria

Died: January 4, 1961, Vienna, Austria

Full name: Erwin Rudolf Josef Alexander Schrödinger

Awards: Nobel Prize in Physics, Max Planck Medal, Austrian Decoration for Science and Art, Erwin Schrödinger Prize

Nationality: Irish, Austrian

Schrödinger equation has the form of a wave equation

$$-\frac{\hbar^2}{2m}\nabla^2\Psi(\boldsymbol{r},t) + V(\boldsymbol{r},t)\Psi(\boldsymbol{r},t) = i\hbar\frac{\partial\Psi(\boldsymbol{r},t)}{\partial t}$$

Schrödinger equation

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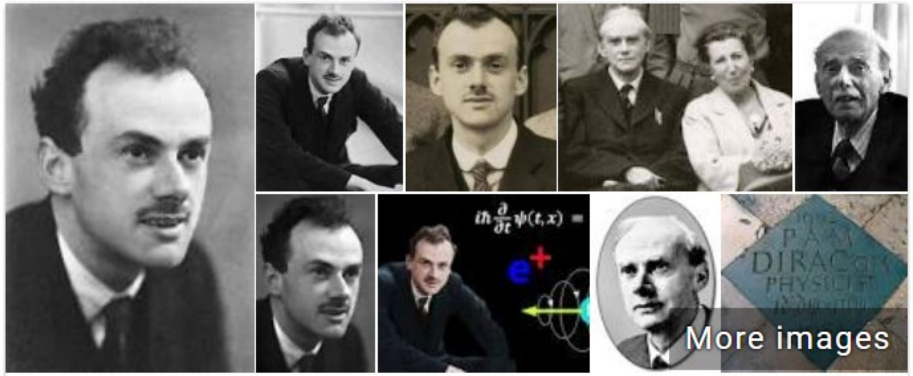
$$-\frac{\hbar^2}{2m}\nabla^2\Psi(\boldsymbol{r},t)+V(\boldsymbol{r},t)\Psi(\boldsymbol{r},t)=i\hbar\frac{\partial\Psi(\boldsymbol{r},t)}{\partial t}$$

Therefore the solution is a linear combination of all the possible wavefunctions

$$\psi(x)=\sum_i c_i\phi_i(x)$$

Dirac notation and wavefunction



Paul Dirac

Physicist

Paul Adrien Maurice Dirac OM FRS was an English theoretical physicist who is regarded as one of the most significant physicists of the 20th century. Dirac made fundamental contributions to the early development of both quantum mechanics and quantum electrodynamics. Wikipedia

Born: August 8, 1902, Bristol, United Kingdom

Died: October 20, 1984, Tallahassee, FL

Field: Theoretical physics

Spouse: Margit Wigner (m. 1937–1984)

Schrödinger equation has the form of a wave equation

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$$\psi(x) = \sum_i c_i\phi_i(x)$$

$$\int_{-\infty}^{+\infty} \phi_j{}^*(x)\ \psi(x)dx = \sum_i c_i \int_{-\infty}^{+\infty} \phi_j(x)^*\ \phi_i(x)dx = c_j\ .$$

In Dirac notation, $|\psi\rangle = \sum_i c_i|\phi_i\rangle$, where $c_j = \langle\phi_j|\psi\rangle$.

Dirac notation and wavefunction

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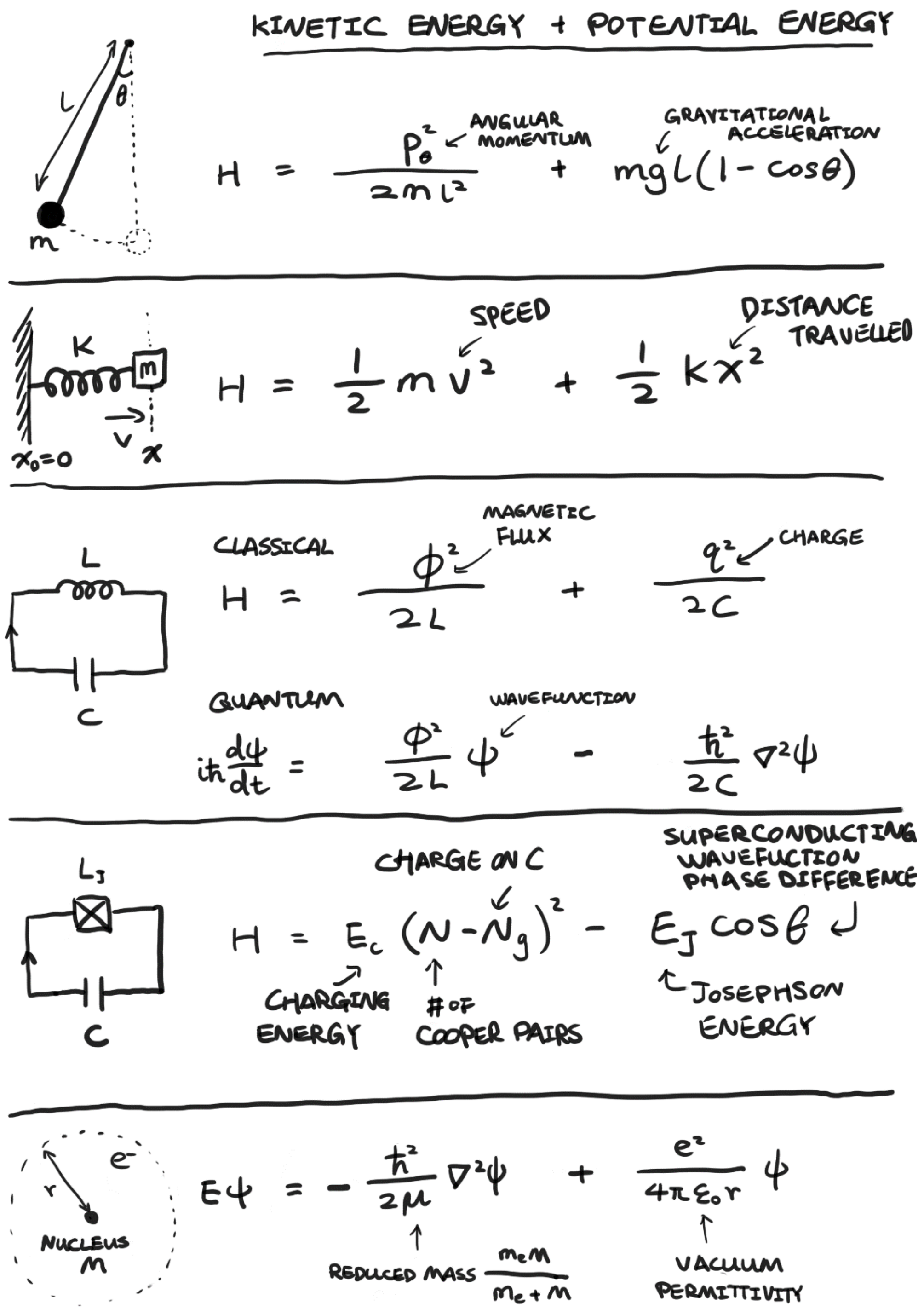
In Dirac notation, $|\psi\rangle = \sum_i c_i|\phi_i\rangle$, where $c_j = \langle\phi_j|\psi\rangle$.

$|\Psi\rangle$ denotes “the state with wavefunction” $\Psi(\boldsymbol{r},t)$

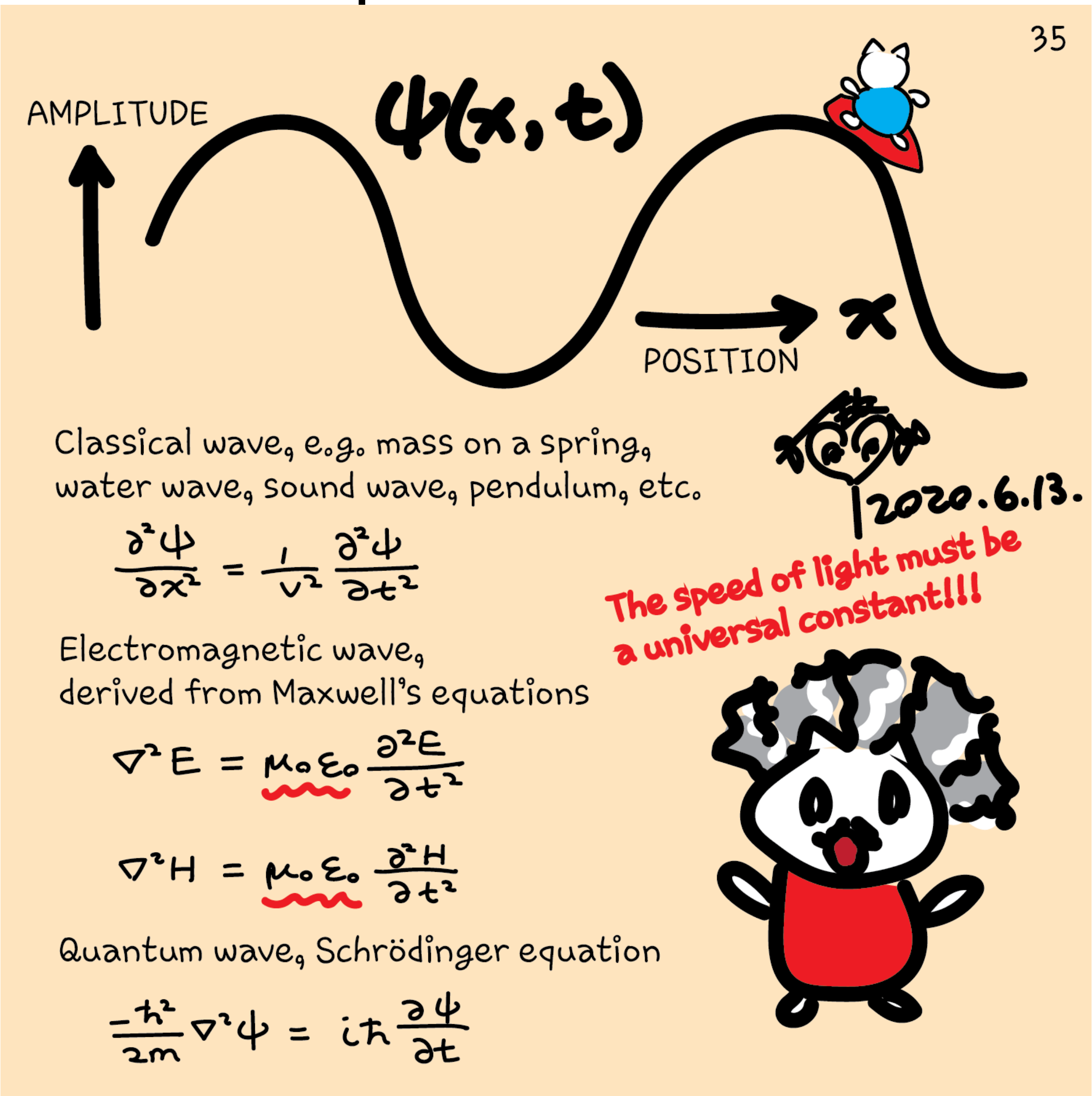
$$\Psi^*(\boldsymbol{r},t) = \langle\Psi|$$

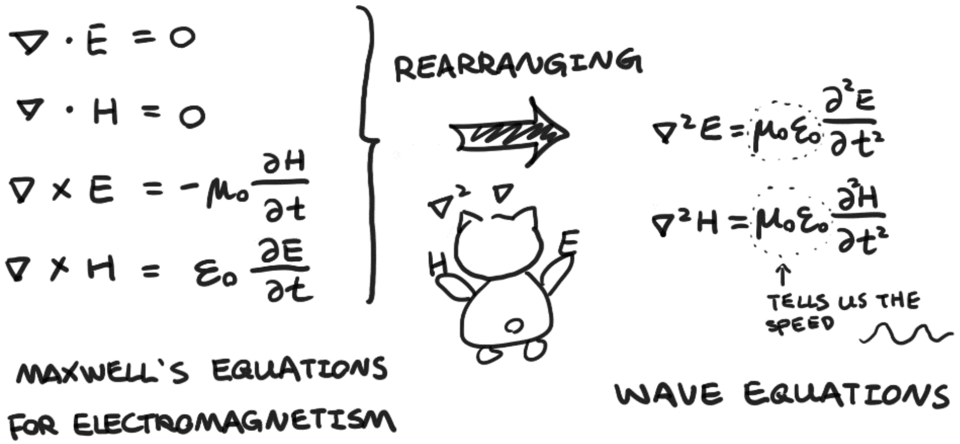
$$\int_{-\infty}^{+\infty} \phi^*(x)\,\psi(x)\,dx \equiv \langle\phi|\psi\rangle$$

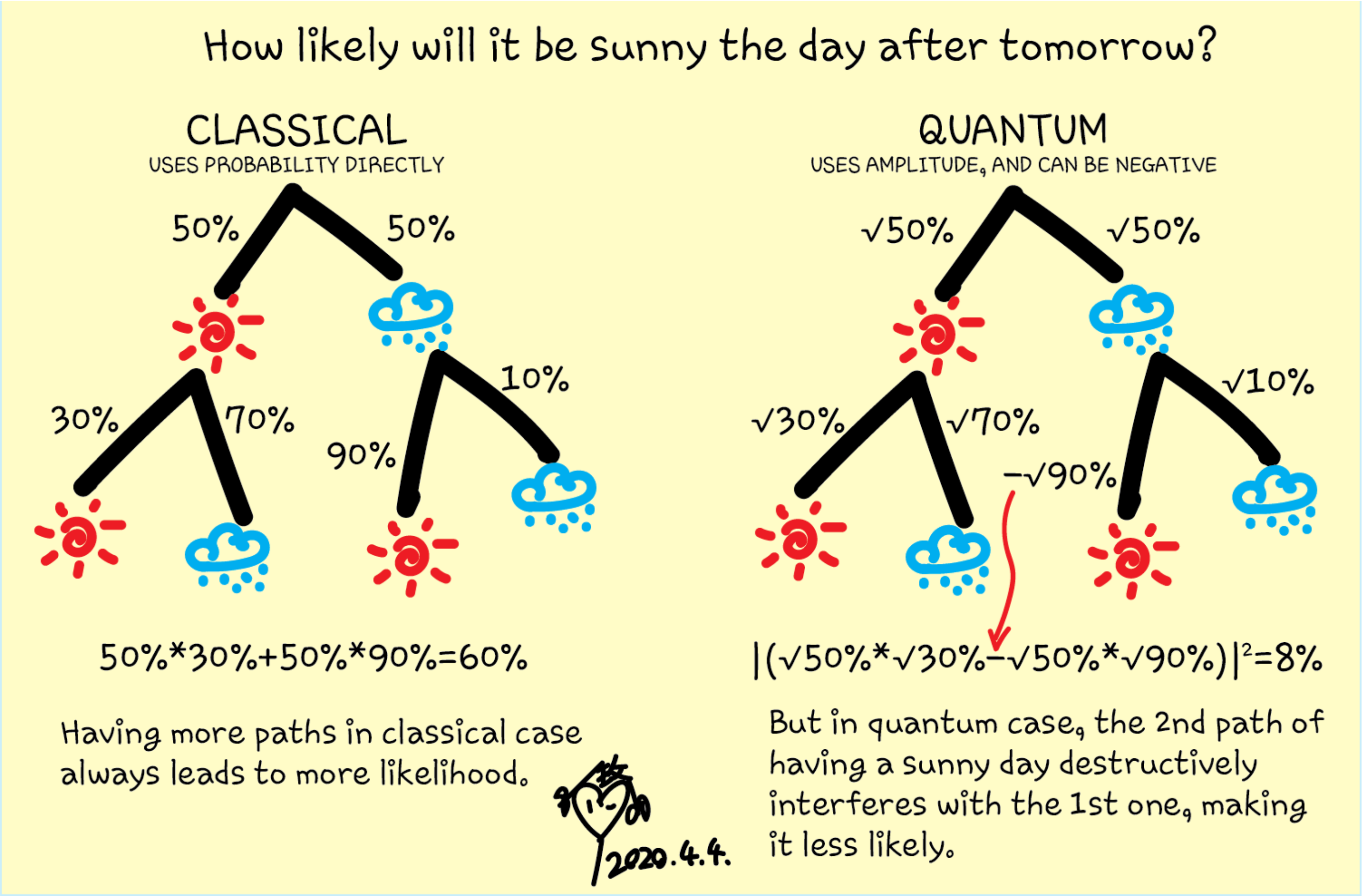
Same behaviours of different systems in our universe



Wave equations



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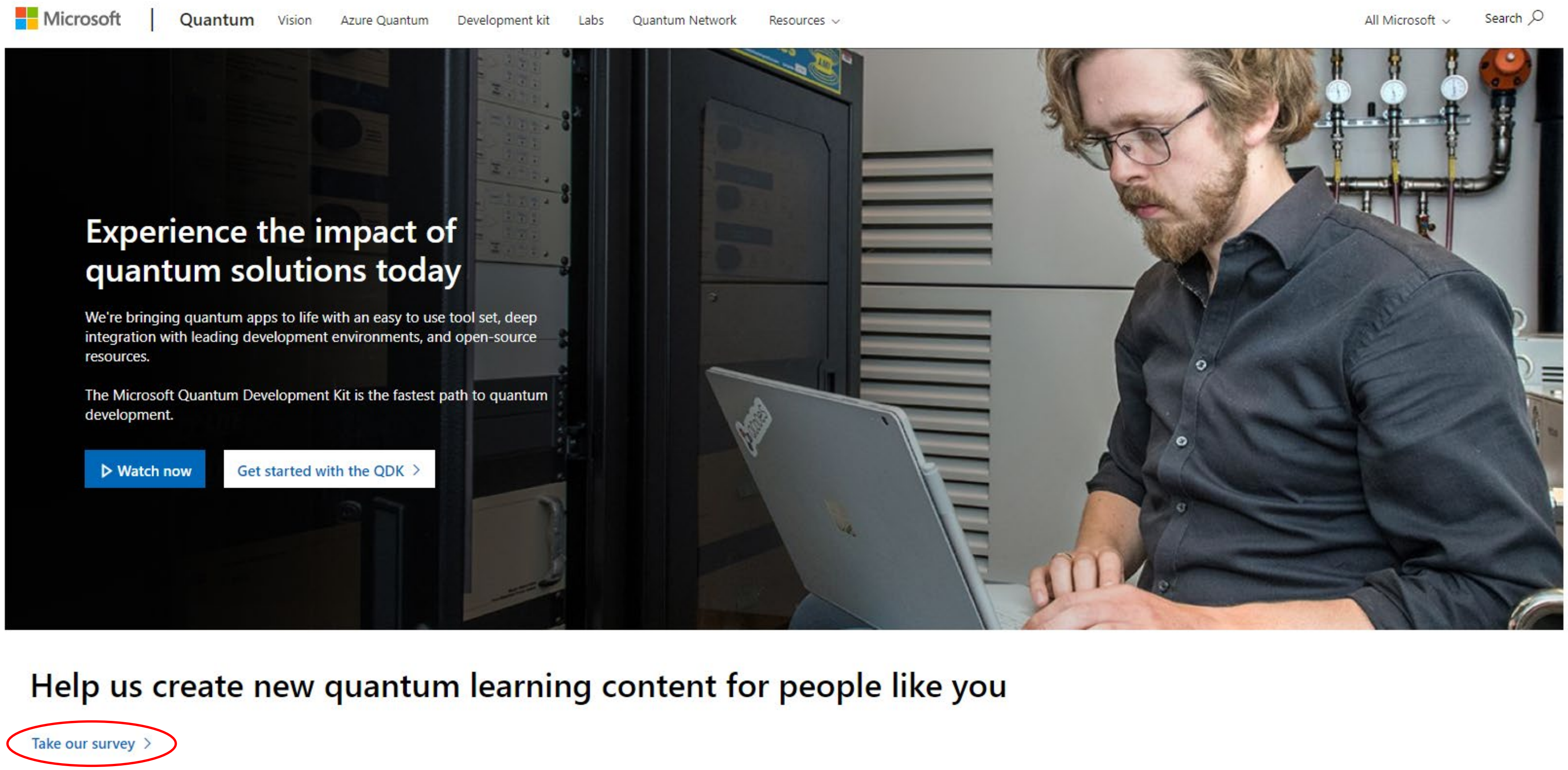

Participate

- Dr. Sarah Kaiser is doing Q# coding live every Wed and Sat at 12pm PT. Check them out! https://www.twitch.tv/crazy4pi314

- Microsoft Q# coding contest is happening from June 19 to June 22, 2020. Register now! https://codeforces.com/blog/entry/77614

Questions

- Post in chat or on Hackaday project https://hackaday.io/project/168554-introduction-to-quantum-computing

- Past Recordings on Hackaday project or my YouTube https://www.youtube.com/c/DrKittyYeung