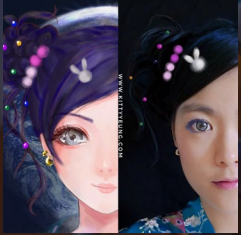


Introduction to Quantum Computing



Kitty Yeung, Ph.D. in Applied Physics

Creative Technologist + Sr. PM
Microsoft

www.artbyphysicistkittyyeung.com



@KittyArtPhysics



@artbyphysicistkittyyeung

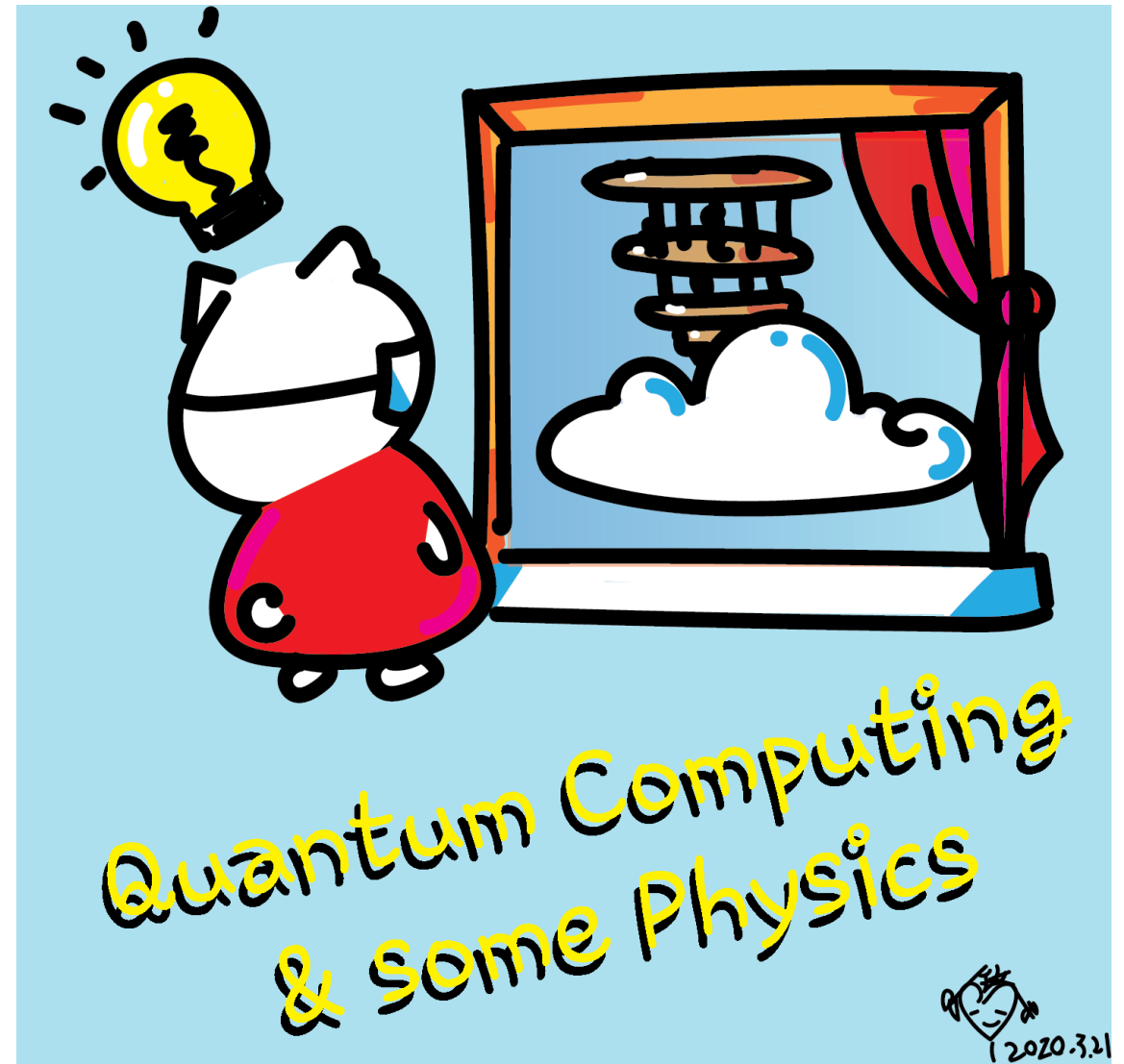
June 14, 2020

Hackaday, session 11

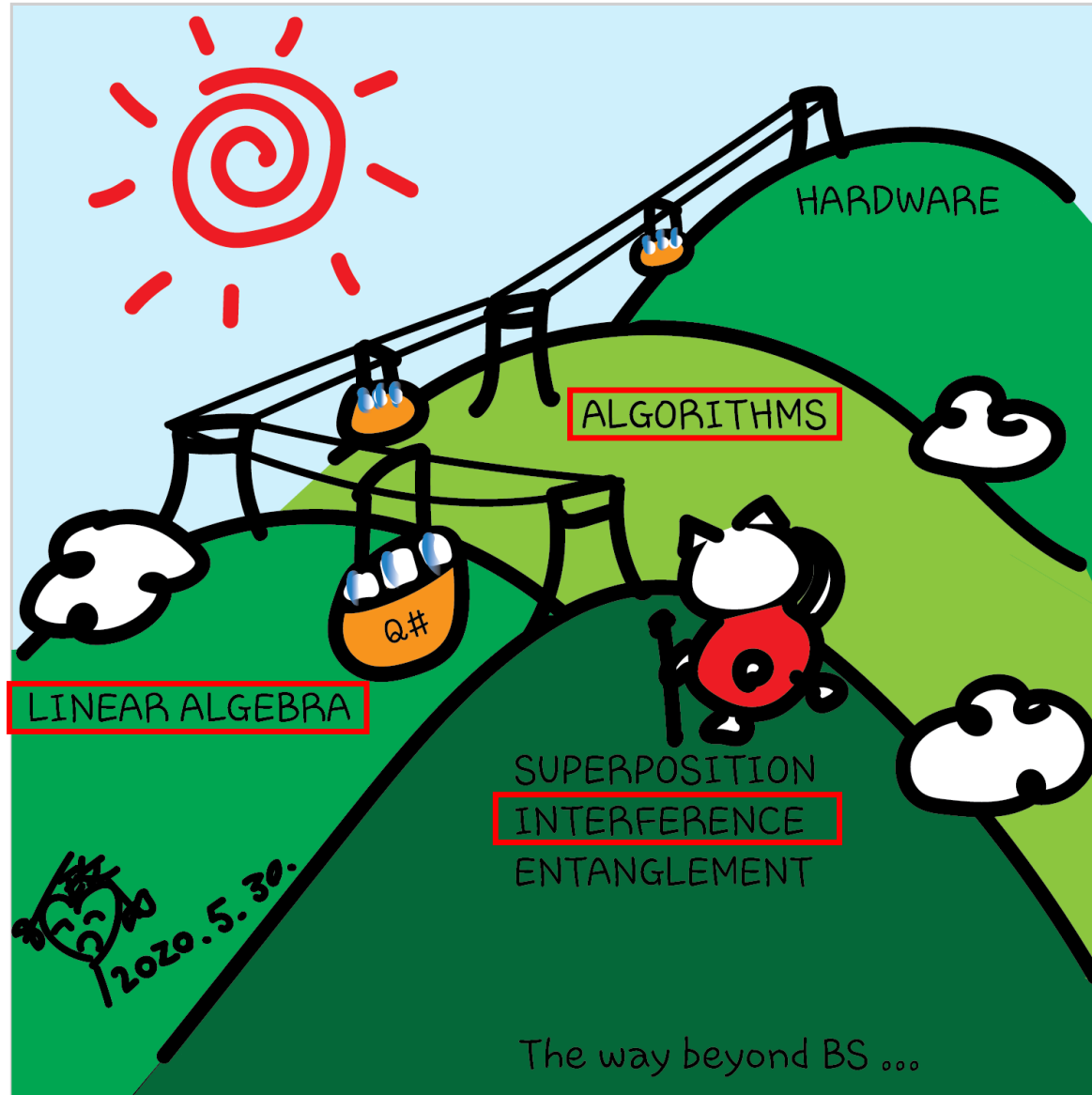
Other communities, session 3

Class structure

- [Comics on Hackaday – Introduction to Quantum Computing](#) every Sun
- 30 mins – 1 hour every Sun, one concept (theory, hardware, programming), Q&A
- Contribute to Q# documentation
<http://docs.microsoft.com/quantum>
- Coding through Quantum Katas
<https://github.com/Microsoft/QuantumKatas/>
- Discuss in Hackaday project comments throughout the week
- Take notes



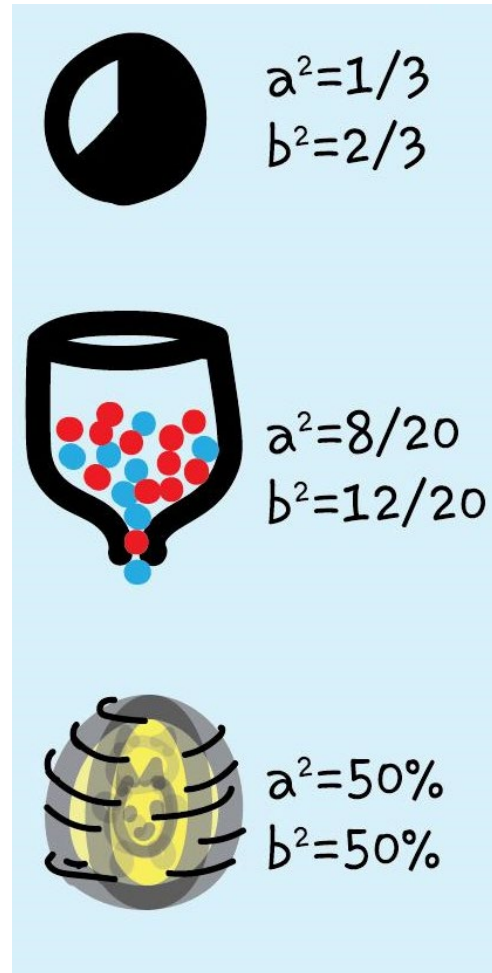
Next class



Qubits & Superposition

$$|\psi\rangle = \begin{pmatrix} a \\ b \end{pmatrix} = a|0\rangle + b|1\rangle$$

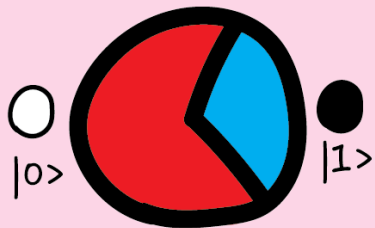
$$|a|^2 + |b|^2 = 1$$



2020.3.28.

A qubit system is all the possible configurations in superposition.

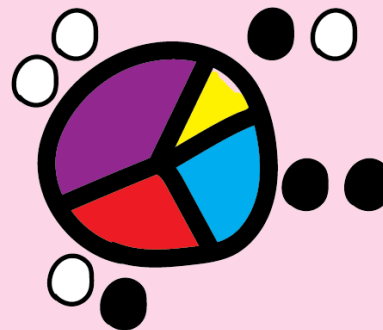
PIE CHART DENOTING PROBABILITY OF EACH CONFIGURATION



ONE QUBIT, TWO CONFIGURATIONS:

$$a|0\rangle + b|1\rangle$$

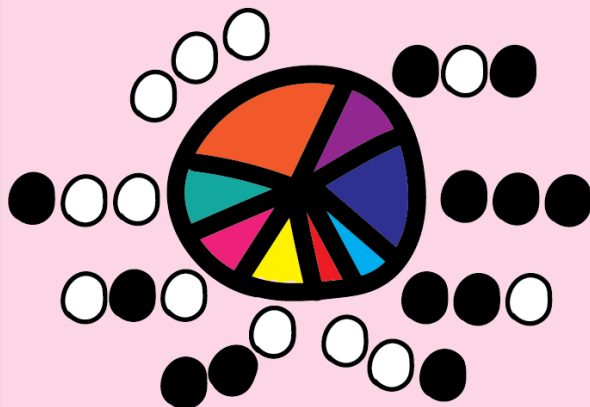
$$a^2 + b^2 = 1 \text{ (total probability adds up to 1)}$$



TWO QUBITS, FOUR CONFIGURATIONS:

$$a|00\rangle + b|01\rangle + c|10\rangle + d|11\rangle$$

$$a^2 + b^2 + c^2 + d^2 = 1$$

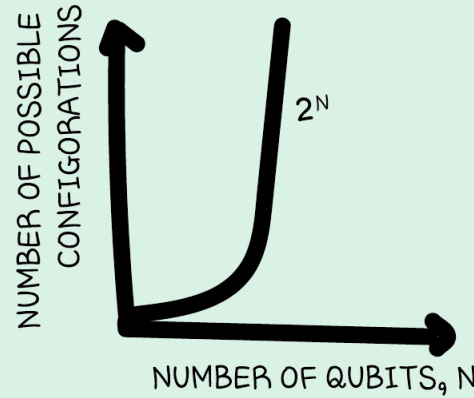


THREE QUBITS, EIGHT CONFIGURATIONS:

$$a|000\rangle + b|001\rangle + c|010\rangle + d|100\rangle + e|110\rangle + f|101\rangle + g|011\rangle + h|111\rangle$$

$$a^2 + b^2 + c^2 + d^2 + e^2 + f^2 + g^2 + h^2 = 1$$

...
N qubits will have 2^N possible configurations in superposition!



Not only does the number of possible configurations grow exponentially with the number of qubits as 2^N , the number of possible combinations of amplitudes is infinite, as long as their squares – the probabilities – add up to 1.

$$a|000\rangle + b|001\rangle + c|010\rangle + d|100\rangle + e|110\rangle + f|101\rangle + g|011\rangle + h|111\rangle$$

THIS SYMBOL MEANS SUMMING ALL N TERMS FROM 1

$$|\psi\rangle = \sum_{i=1}^N c_i |\psi_i\rangle$$

EACH POSSIBLE CONFIGURATION

AN N-QUBIT STATE

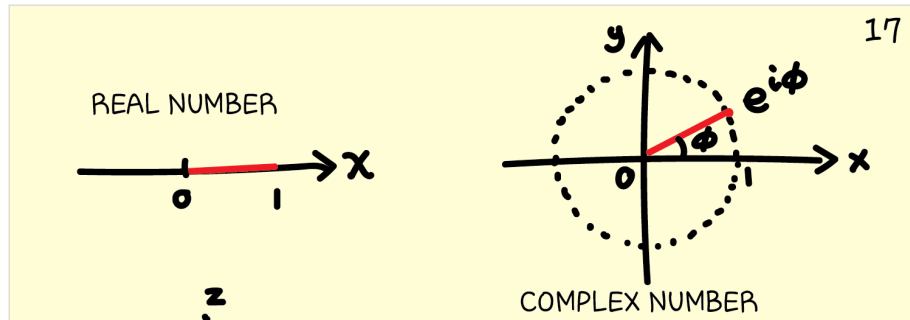
NATURE DOES PLAY DICE!!!



The amplitude $c_i = a, b, c, d \dots n$ can be positive numbers $1, 1/2, 1/3, 1/4 \dots n$ or negative numbers $-1, -1/2, -1/3, -1/4 \dots n$ (these are real numbers) or imaginary numbers $(+/-) i, 1/2i, 1/3i, 1/4i \dots ni$ or 0. In general they can be complex numbers (with real and imaginary parts with positive or negative signs)!

What's the consequence?

Complex numbers



A one-qubit state $|\psi\rangle = z_0|0\rangle + z_1|1\rangle$ can be written as

$$(a_0 + b_0)|0\rangle + (a_1 + b_1)|1\rangle$$

$$= r_0 e^{i\varphi_0} |0\rangle + r_1 e^{i\varphi_1} |1\rangle.$$

Therefore, the probability of finding state $|0\rangle$ is $|r_0 e^{i\varphi_0}|^2 = r_0^2$. Similarly, for state $|1\rangle$ it is r_1^2 . The probability is determined by the magnitude of amplitude and is independent from phase.

34

Any complex number can be expressed by a real part and an imaginary part.

$$Z = a + ib \quad i = \sqrt{-1}$$

$= r e^{i\phi}$

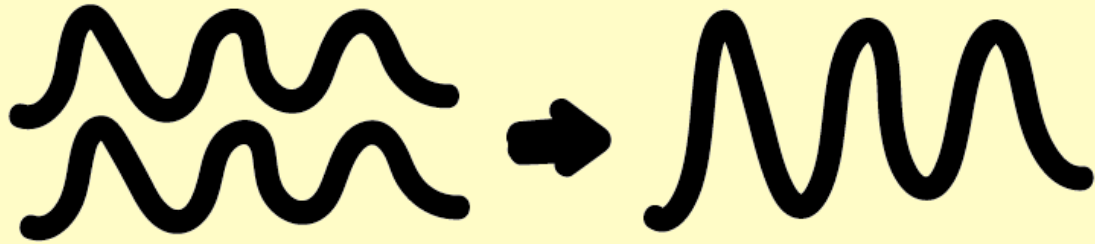
There are two ways to express the same number, z : in Cartesian coordinates with a and b , or in polar coordinates with r and Φ .

$r = \sqrt{a^2 + b^2}$ is the magnitude of z ;
 Φ , with $e^{i\Phi} = \cos \Phi + i \sin \Phi$, is the phase of z .

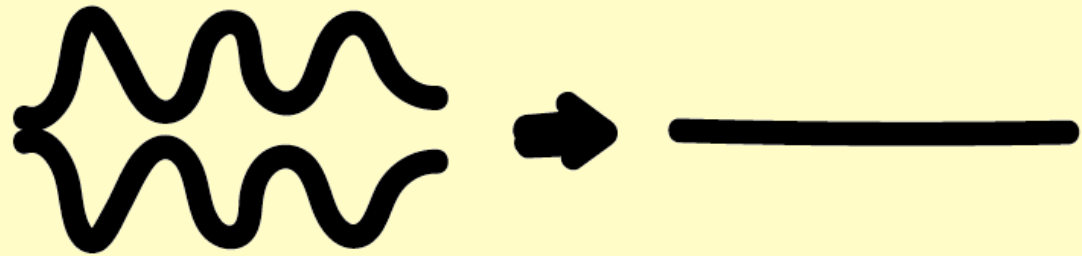
If z is an amplitude of a quantum state, the probability of the state is the magnitude squared: $|r e^{i\Phi}|^2 = r^2$.

COMPLEX NUMBER

Our daily experience of amplitudes (like those of water waves, light waves, sound waves, etc.) has told us:



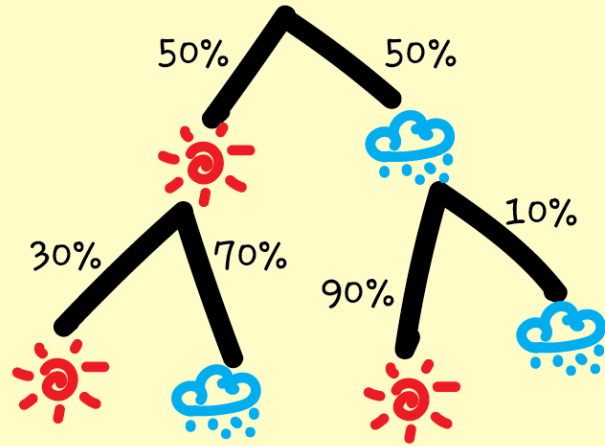
AMPLITUDES CAN ADD UP =
CONSTRUCTIVE INTERFERECE



AMPLITUDES CAN CANCEL OUT =
DESTRUCTIVE INTERFERENCE

How likely will it be sunny the day after tomorrow?

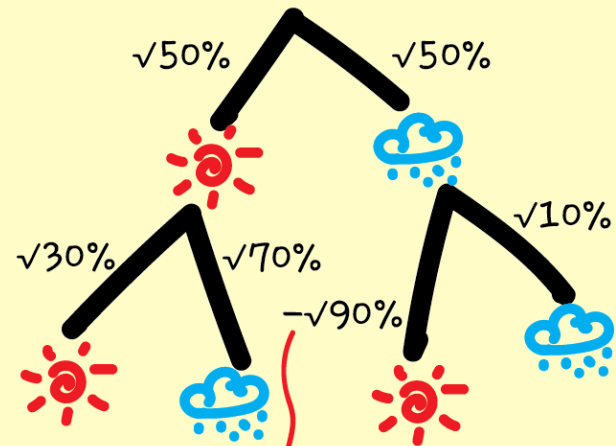
CLASSICAL
USES PROBABILITY DIRECTLY



$$50\% * 30\% + 50\% * 90\% = 60\%$$

Having more paths in classical case always leads to more likelihood.

QUANTUM
USES AMPLITUDE, AND CAN BE NEGATIVE



$$|(\sqrt{50\%} * \sqrt{30\%} - \sqrt{50\%} * \sqrt{90\%})|^2 = 8\%$$

But in quantum case, the 2nd path of having a sunny day destructively interferes with the 1st one, making it less likely.


2020.4.4.

Generalized probability theory

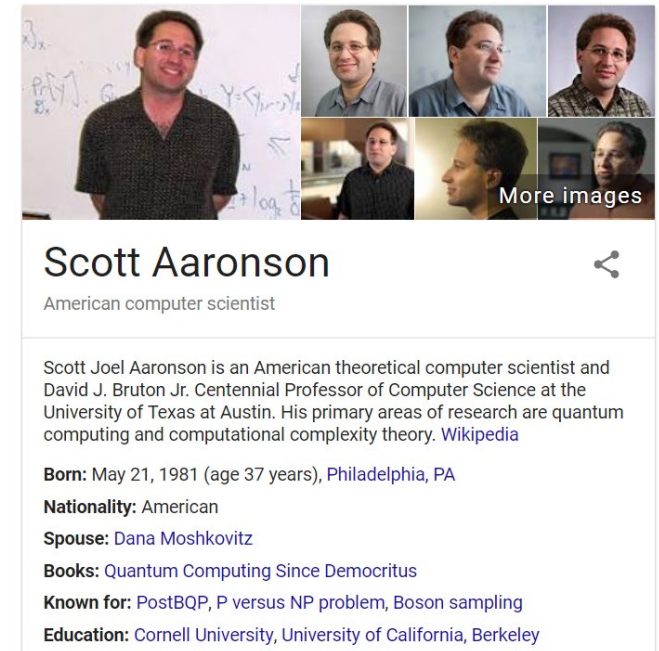
$$\sum_i p_i = 1$$

1-norm
Classical

$$\sum_i |a_i|^2 = 1$$

2-norm
Quantum mechanical

Amplitude can be positive, negative or complex



2-norm Vs 1-norm

<https://www.scottaaronson.com/democritus/lec9.html>

To read more rigorous mathematical derivations of the axioms in modern quantum theory:

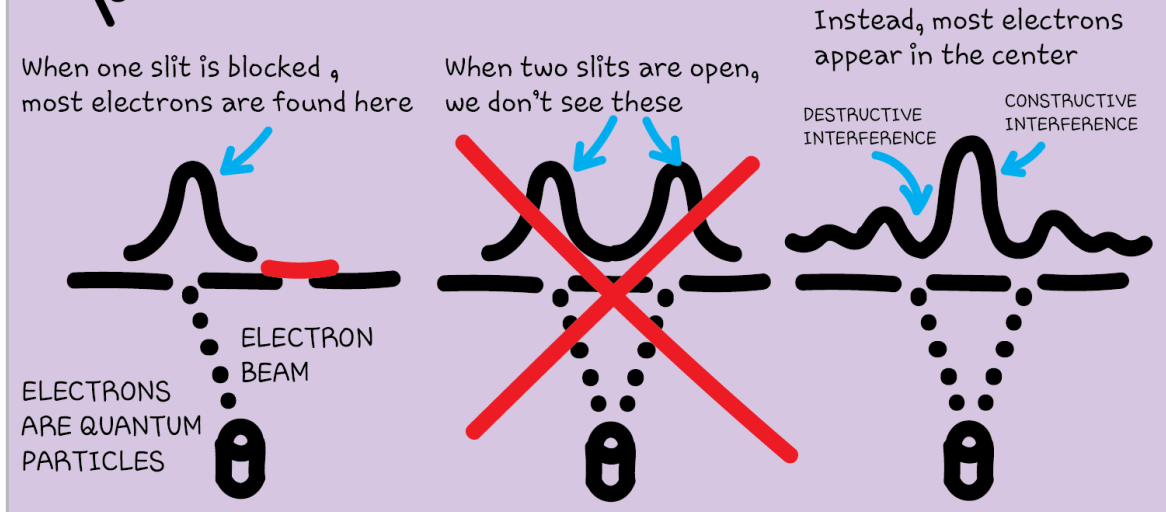
- <https://arxiv.org/abs/quant-ph/0101012>
- <https://arxiv.org/abs/1011.6451>
- <https://arxiv.org/abs/quant-ph/0104088>



So, the things we observe (measure) are the results of interference. Possible results from constructive interference are more likely to be measured. The other possibilities cancel each other out through destructive interference.

2020.4.5.

The famous double-slit experiment is a direct manifestation of quantum interference.



Interference is one of the "strange" behaviours of quantum systems enabled by superposition. What else?

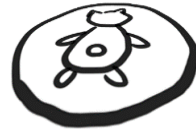
Measurement

BOTH HEAD AND TAIL
ARE POSSIBLE



MEASUREMENT

ONLY ONE OUTCOME
CANNOT RETURN
TO PREVIOUS STATE



Not reversible

$$|\psi\rangle = c_{00}|00\rangle + c_{01}|01\rangle + c_{10}|10\rangle + c_{11}|11\rangle$$

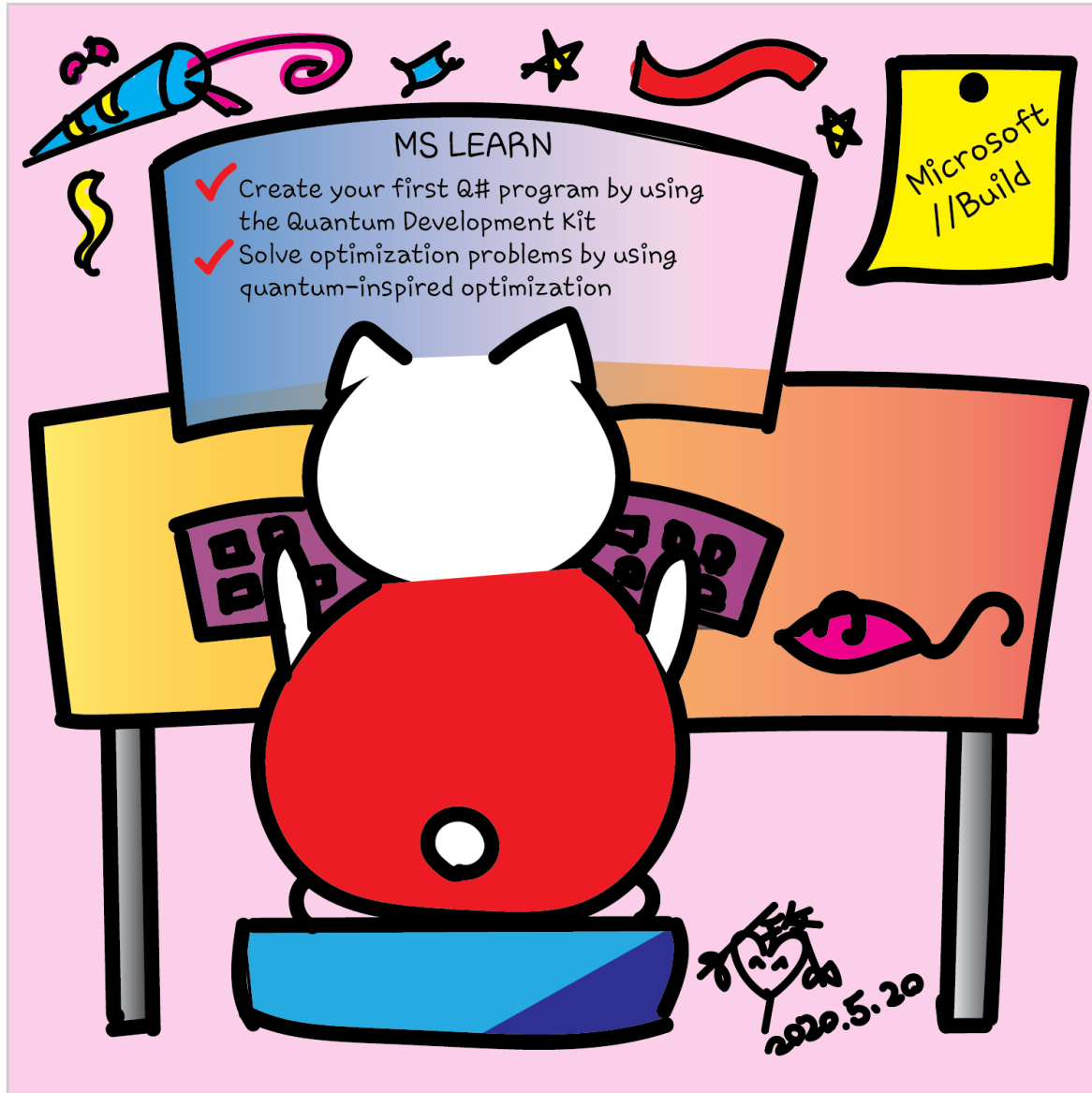
$$P = |c_{00}|^2 + |c_{01}|^2$$

If first qubit is 0

$$|\psi'\rangle = \frac{c_{00}|00\rangle + c_{01}|01\rangle}{\sqrt{P}}$$

After measurement

aka.ms/learnqc



Create your first Q# program by using the Quantum Development Kit

48 min • Module • 8 Units

★★★★★ 4.9 (48)

Beginner Developer Quantum Development Kit Quantum

Get started with Q# programming by building a quantum random number generator.

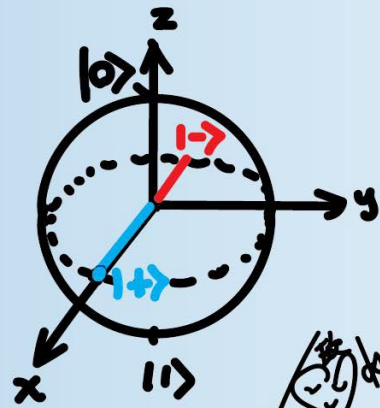
900 XP

<https://docs.microsoft.com/en-us/learn/modules/qsharp-create-first-quantum-development-kit/>

Q# exercise:

Option 1: No installation, web-based Jupyter Notebooks

- The Quantum Katas project (tutorials and exercises for learning quantum computing) <https://github.com/Microsoft/QuantumKatas>
- **Measurement**
- Tasks 1.1-1.4



Another important gate is the H 20
(or Hadamard) gate. It changes states $|0\rangle$ and $|1\rangle$ and creates two new states in between them:

$$H|0\rangle = |+\rangle = (|0\rangle + |1\rangle) / \sqrt{2}$$

$$H|1\rangle = |-\rangle = (|0\rangle - |1\rangle) / \sqrt{2}$$

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}$$

General rotation

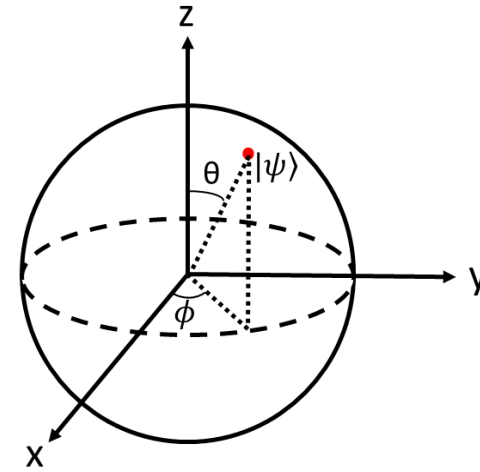
In general, rotation gates, R , about an axis can be described by the angles ϕ and θ :

$$R_z(\phi) = \begin{bmatrix} e^{-i\phi/2} & 0 \\ 0 & e^{i\phi/2} \end{bmatrix},$$

$$R_y(\theta) = \begin{bmatrix} \cos \frac{\theta}{2} & -\sin \frac{\theta}{2} \\ \sin \frac{\theta}{2} & \cos \frac{\theta}{2} \end{bmatrix},$$

and

$$R_x(\theta) = \begin{bmatrix} \cos \frac{\theta}{2} & -i \sin \frac{\theta}{2} \\ -i \sin \frac{\theta}{2} & \cos \frac{\theta}{2} \end{bmatrix} \\ = R_z\left(\frac{\pi}{2}\right) R_y(\theta) R_z\left(-\frac{\pi}{2}\right).$$



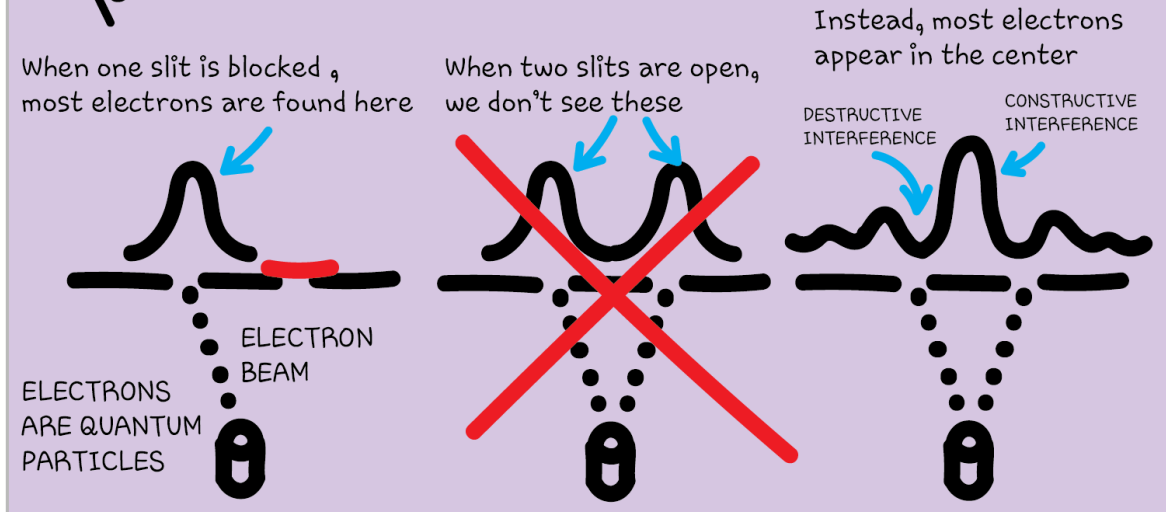
<https://review.docs.microsoft.com/en-us/quantum/concepts/the-qubit?branch=tensor-product>



So, the things we observe (measure) are the results of interference. Possible results from constructive interference are more likely to be measured. The other possibilities cancel each other out through destructive interference.

2020.4.5.

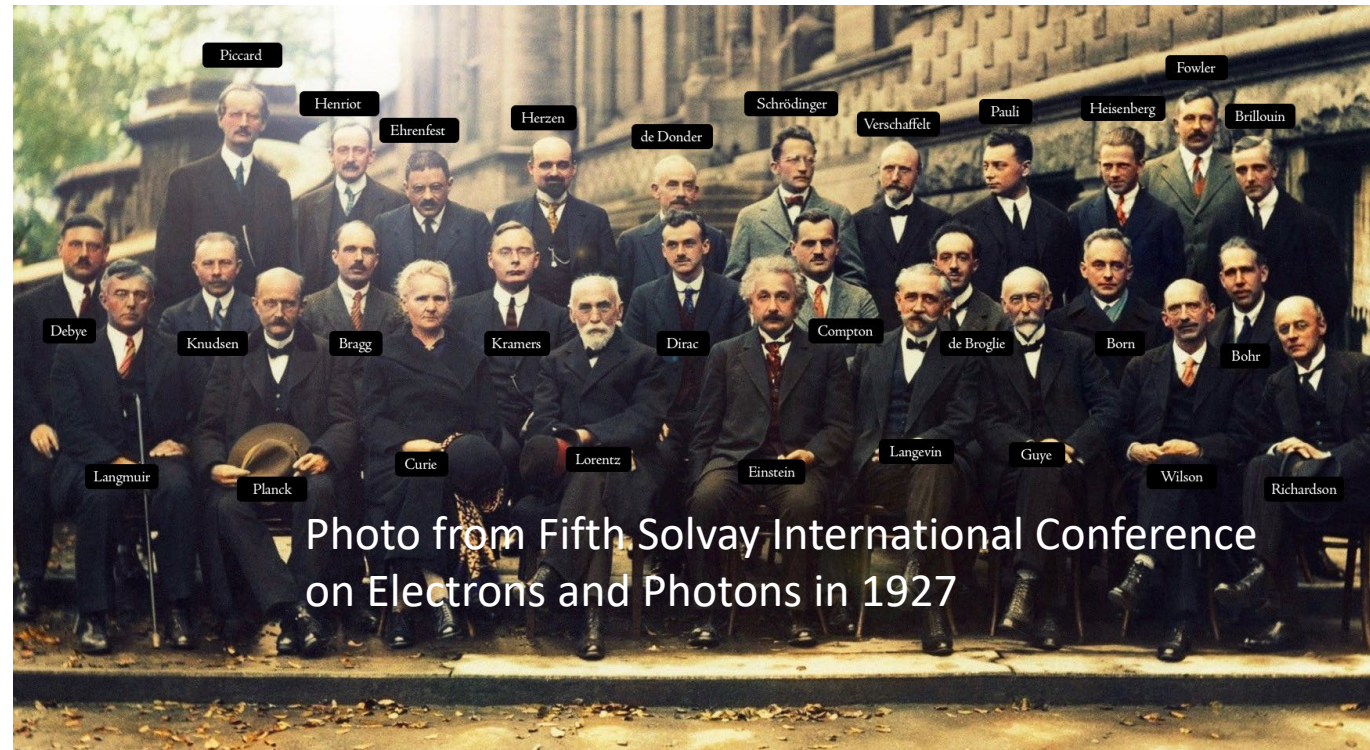
The famous double-slit experiment is a direct manifestation of quantum interference.



Interference is one of the "strange" behaviours of quantum systems enabled by superposition. What else?

Development of quantum mechanics (~100 years ago)

- Quantization of energy (black-body radiation – the UV catastrophe, the photoelectric effect, the Compton effect, Stern-Gerlach experiment)
- Wave-particle duality (double-slit experiment, atomic structure, de Broglie hypothesis, electron diffraction, molecular diffraction)
- Schrödinger equation



Schrödinger equation



Erwin Schrödinger

Austrian-Irish physicist

Erwin Rudolf Josef Alexander Schrödinger, sometimes written as Erwin Schrodinger or Erwin Schroedinger, was a Nobel Prize-winning Austrian-Irish physicist who developed a number of fundamental results ... [Wikipedia](#)

Born: August 12, 1887, Erdberg, Vienna, Austria

Died: January 4, 1961, Vienna, Austria

Full name: Erwin Rudolf Josef Alexander Schrödinger

Awards: Nobel Prize in Physics, Max Planck Medal, Austrian Decoration for Science and Art, Erwin Schrödinger Prize

Nationality: Irish, Austrian

Schrödinger equation has the form of a wave equation

$$-\frac{\hbar^2}{2m}\nabla^2\Psi(\mathbf{r},t) + V(\mathbf{r},t)\Psi(\mathbf{r},t) = i\hbar\frac{\partial\Psi(\mathbf{r},t)}{\partial t}$$

Schrödinger equation

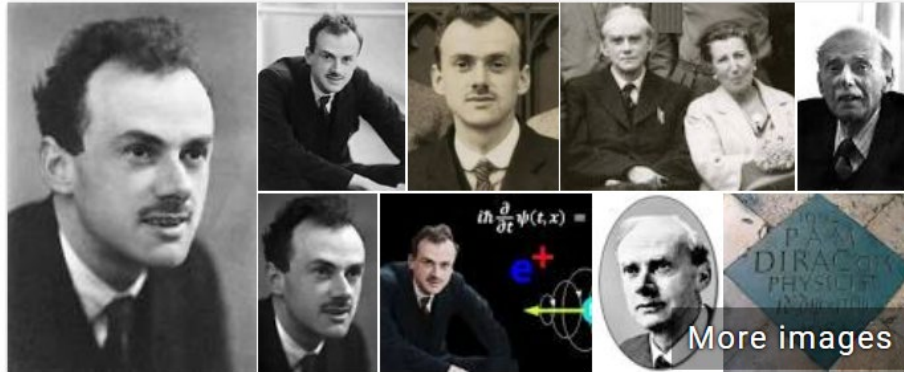
Schrödinger equation has the form of a wave equation

$$-\frac{\hbar^2}{2m}\nabla^2\Psi(\mathbf{r},t) + V(\mathbf{r},t)\Psi(\mathbf{r},t) = i\hbar\frac{\partial\Psi(\mathbf{r},t)}{\partial t}$$

Therefore the solution
is a linear combination
of all the possible
wavefunctions

$$\psi(x) = \sum_i c_i \phi_i(x)$$

Dirac notation and wavefunction



Paul Dirac

Physicist

Paul Adrien Maurice Dirac OM FRS was an English theoretical physicist who is regarded as one of the most significant physicists of the 20th century. Dirac made fundamental contributions to the early development of both quantum mechanics and quantum electrodynamics. [Wikipedia](#)

Born: August 8, 1902, Bristol, United Kingdom

Died: October 20, 1984, Tallahassee, FL

Field: Theoretical physics

Spouse: Margit Wigner (m. 1937–1984)

Schrödinger equation has the form of a wave equation

$$-\frac{\hbar^2}{2m} \nabla^2 \Psi(\mathbf{r}, t) + V(\mathbf{r}, t) \Psi(\mathbf{r}, t) = i\hbar \frac{\partial \Psi(\mathbf{r}, t)}{\partial t}$$

$$\psi(x) = \sum_i c_i \phi_i(x)$$

$$\int_{-\infty}^{+\infty} \phi_j^*(x) \psi(x) dx = \sum_i c_i \int_{-\infty}^{+\infty} \phi_j(x)^* \phi_i(x) dx = c_j .$$

In Dirac notation, $|\psi\rangle = \sum_i c_i |\phi_i\rangle$, where $c_j = \langle \phi_j | \psi \rangle$.

Dirac notation and wavefunction

Schrödinger equation has the form of a wave equation

$$-\frac{\hbar^2}{2m} \nabla^2 \Psi(\mathbf{r}, t) + V(\mathbf{r}, t) \Psi(\mathbf{r}, t) = i\hbar \frac{\partial \Psi(\mathbf{r}, t)}{\partial t}$$

$$\psi(x) = \sum_i c_i \phi_i(x)$$

$$\int_{-\infty}^{+\infty} \phi_j^*(x) \psi(x) dx = \sum_i c_i \int_{-\infty}^{+\infty} \phi_j(x)^* \phi_i(x) dx = c_j.$$

In Dirac notation, $|\psi\rangle = \sum_i c_i |\phi_i\rangle$, where $c_j = \langle \phi_j | \psi \rangle$.

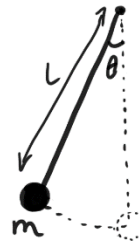
$|\Psi\rangle$ denotes “the state with wavefunction” $\Psi(\mathbf{r}, t)$

$$\Psi^*(\mathbf{r}, t) = \langle \Psi |$$

$$\int_{-\infty}^{+\infty} \phi^*(x) \psi(x) dx \equiv \langle \phi | \psi \rangle$$

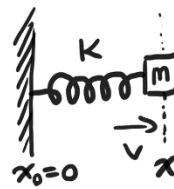
Same behaviours of different systems in our universe

KINETIC ENERGY + POTENTIAL ENERGY



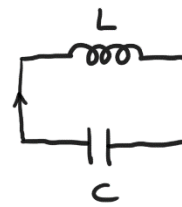
$$H = \frac{p_\theta^2}{2mL^2} + mgL(1 - \cos\theta)$$

ANGULAR MOMENTUM GRAVITATIONAL ACCELERATION



$$H = \frac{1}{2} m v^2 + \frac{1}{2} k x^2$$

SPEED DISTANCE TRAVELLED



CLASSICAL

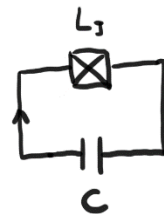
$$H = \frac{\phi^2}{2L} + \frac{q^2}{2C}$$

MAGNETIC FLUX CHARGE

QUANTUM

$$i\hbar \frac{d\psi}{dt} = \frac{\phi^2}{2L} \psi - \frac{\hbar^2}{2C} \nabla^2 \psi$$

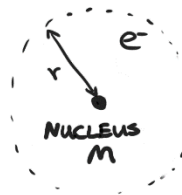
WAVEFUNCTION



CHARGE ON C SUPERCONDUCTING WAVEFUNCTION PHASE DIFFERENCE

$$H = E_C (N - N_g)^2 - E_J \cos\theta$$

CHARGING ENERGY # OF COOPER PAIRS JOSEPHSON ENERGY

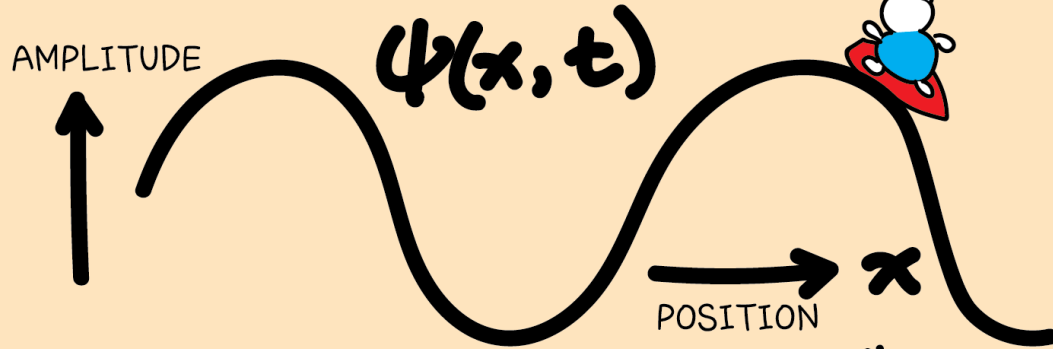


$$E\psi = - \frac{\hbar^2}{2\mu} \nabla^2 \psi + \frac{e^2}{4\pi\epsilon_0 r} \psi$$

REDUCED MASS $\frac{m_e M}{m_e + M}$ VACUUM PERMITTIVITY

Wave equations

35



Classical wave, e.g. mass on a spring, water wave, sound wave, pendulum, etc.

$$\frac{\partial^2 \psi}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 \psi}{\partial t^2}$$

Electromagnetic wave, derived from Maxwell's equations

$$\nabla^2 E = \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2}$$

$$\nabla^2 H = \mu_0 \epsilon_0 \frac{\partial^2 H}{\partial t^2}$$

Quantum wave, Schrödinger equation

$$-\frac{\hbar^2}{2m} \nabla^2 \psi = i\hbar \frac{\partial \psi}{\partial t}$$

The speed of light must be a universal constant!!!



2020.6.13.

$$\nabla \cdot E = 0$$

$$\nabla \cdot H = 0$$

$$\nabla \times E = -\mu_0 \frac{\partial H}{\partial t}$$

$$\nabla \times H = \epsilon_0 \frac{\partial E}{\partial t}$$

MAXWELL'S EQUATIONS FOR ELECTROMAGNETISM

REARRANGING



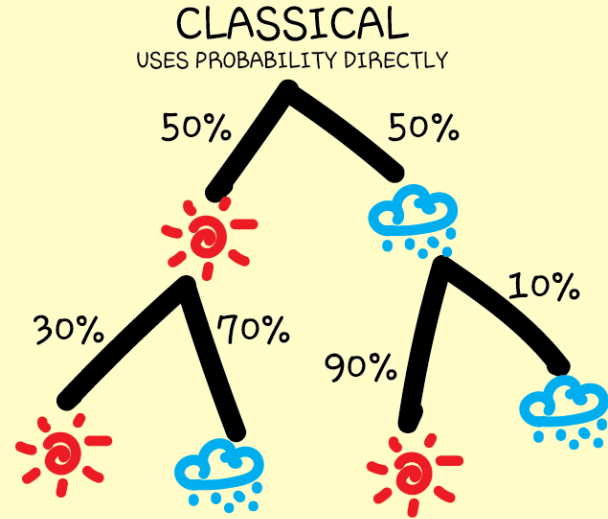
$$\nabla^2 E = \mu_0 \epsilon_0 \frac{\partial^2 E}{\partial t^2}$$

$$\nabla^2 H = \mu_0 \epsilon_0 \frac{\partial^2 H}{\partial t^2}$$

↑ TELLS US THE SPEED

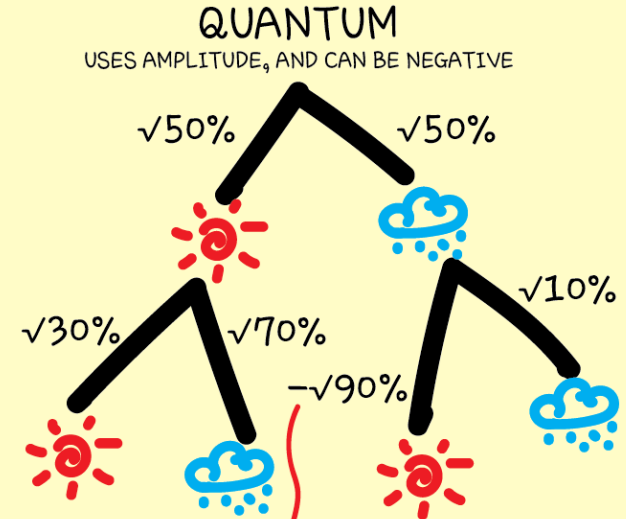
WAVE EQUATIONS

How likely will it be sunny the day after tomorrow?



$$50\% * 30\% + 50\% * 90\% = 60\%$$

Having more paths in classical case always leads to more likelihood.

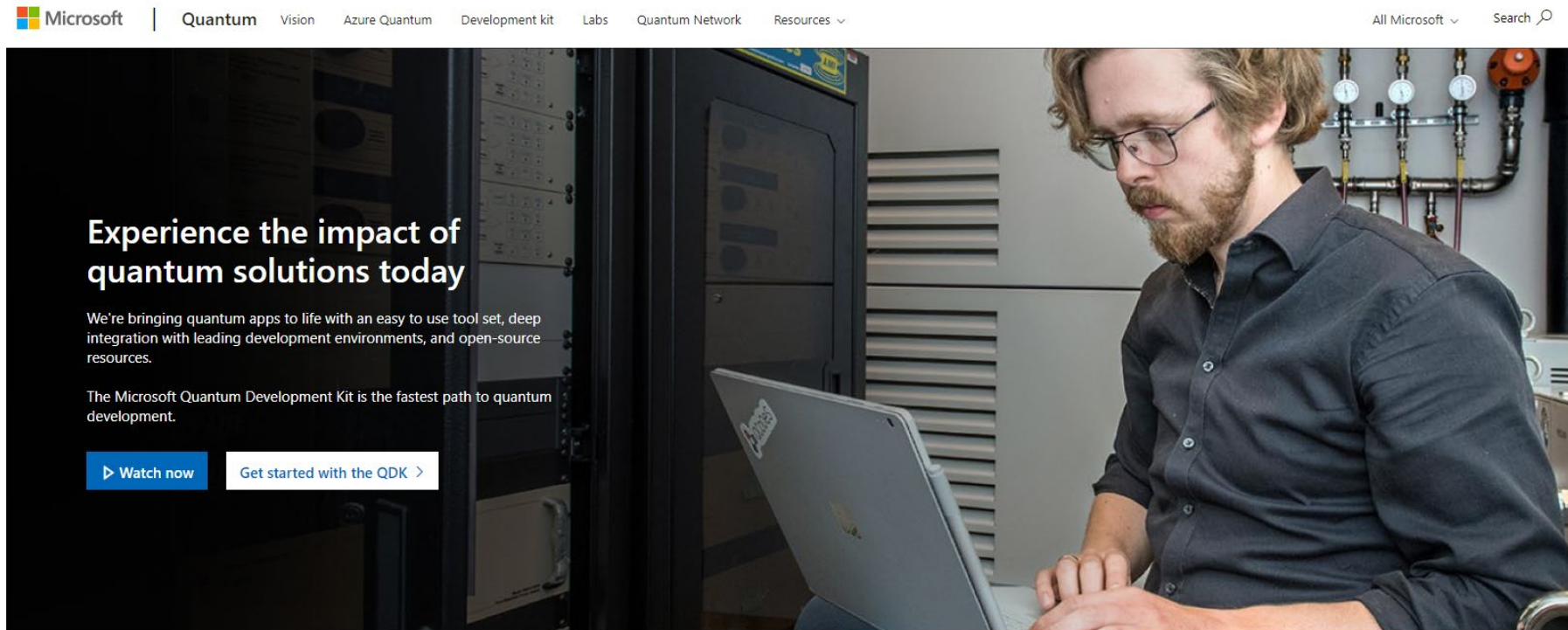


$$|(\sqrt{50\%} * \sqrt{30\%} - \sqrt{50\%} * \sqrt{90\%})|^2 = 8\%$$

But in quantum case, the 2nd path of having a sunny day destructively interferes with the 1st one, making it less likely.


2020.4.4.

<https://www.microsoft.com/quantum/development-kit>

The image shows the Microsoft Quantum Development Kit landing page. At the top, there is a navigation bar with the Microsoft logo on the left and links for Quantum, Vision, Azure Quantum, Development kit, Labs, Quantum Network, and Resources. On the right side of the navigation bar, there are links for 'All Microsoft' and a search icon. The main content area features a large background image of a man with a beard and glasses, wearing a dark blue shirt, sitting at a desk and working on a laptop. To the left of the man, there is a server rack. The text on the page reads: 'Experience the impact of quantum solutions today'. Below this, it says: 'We're bringing quantum apps to life with an easy to use tool set, deep integration with leading development environments, and open-source resources.' Further down, it states: 'The Microsoft Quantum Development Kit is the fastest path to quantum development.' At the bottom of the main content area, there are two buttons: a blue button with a play icon and the text 'Watch now', and a white button with the text 'Get started with the QDK >'.

Microsoft | Quantum Vision Azure Quantum Development kit Labs Quantum Network Resources

All Microsoft Search

Experience the impact of quantum solutions today

We're bringing quantum apps to life with an easy to use tool set, deep integration with leading development environments, and open-source resources.

The Microsoft Quantum Development Kit is the fastest path to quantum development.

[▶ Watch now](#) [Get started with the QDK >](#)

Help us create new quantum learning content for people like you

[Take our survey >](#)

Participate

- Dr. Sarah Kaiser is doing Q# coding live every Wed and Sat at 12pm PT. Check them out! <https://www.twitch.tv/crazy4pi314>
- Microsoft Q# coding contest is happening from June 19 to June 22, 2020. Register now! <https://codeforces.com/blog/entry/77614>

Questions

- Post in chat or on Hackaday project
<https://hackaday.io/project/168554-introduction-to-quantum-computing>
- Past Recordings on Hackaday project or my YouTube
<https://www.youtube.com/c/DrKittyYeung>